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VOL. 6

# SIGHT REDUCTION TABLES

FOR

# MARINE NAVIGATION

## LATITUDES 75°—90°, Inclusive

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# PREFACE

Many requirements exist for the solution of the general astronomical or navigational triangle, for which the navigator's needs are foremost; for upon its accurate and rapid solution depends his effectiveness in locating himself expeditiously upon the Sumner Line of Position. This six-volume series of *Sight Reduction Tables for Marine Navigation* is designed to effect all solutions of the navigational triangle, having given two sides and the included angle, to find a third side and adjacent angle. A simple method is suggested for extracting from the tables the numerical value of the parallactic or position angle, thus completing the triangle solution by providing the sixth element. The tabular data are arranged to facilitate rapid position-determination utilizing the Marcq Saint Hilaire or intercept method.

Tabulations for entering arguments of integral degrees of latitude, declination, and local hour angle include computed values of altitude, successive altitude differences with sign, and azimuth angle. The data are applicable to the solution of sights of all celestial bodies, for latitude and declination of both same and contrary name. Altitudes and their differences are tabulated to the nearest tenth of a minute, azimuth angles to the nearest tenth of a degree. The tables, intended for use with *The Nautical Almanac*, are designed for precise interpolation of altitude for declination only, utilizing convenient interpolation tables which facilitate linear interpolation and provide additionally for the effect of second differences when required. Interpolation of altitude for latitude and local hour angle may also be made by employing the diagrams provided.

The primary table entry is local hour angle—not meridian angle—which is measured westward through 360° from the local meridian to the hour circle of the body. Each volume covers 16° of latitude and is arranged in two zones of 8° each. All latitudes of a zone appear as horizontal entries above each column on every page of that section of the volume, the first degree of each volume overlapping the last degree of the preceding volume. All declinations are given as vertical arguments on each page. Altitudes are tabulated from the horizon to the zenith. Azimuth angles are tabulated through 180°, and rules appear on each opening of the tables for converting azimuth angle to true azimuth. Full explanation of the tables' concept, format, arrangement, and method of interpolation is given in the Introduction. Included examples illustrate methods and techniques employed. Peculiarities of the tables and their application to the solution of certain navigational problems are also covered. Standard navigational manuals should be consulted when more detailed information on line of position methods and techniques is desired.

These new tables, although similar to H.O. Pub. No. 214 (H.D. 486 in the United Kingdom), *Tables of Computed Altitude and Azimuth*, provide full coverage, and are compact in design, format and arrangement. Their completeness and utility suggest their possible broad application to the solution of many space-age problems in closely associated fields of science and technology. For air navigation the H.O. Pub. No. 249 (A.P. 3270 in the United Kingdom) series of tables, consisting of three volumes and entitled *Sight Reduction Tables for Air Navigation*, is recommended.

The concept, design, development, and preparation of these tables are the results of the collaborative efforts and joint accomplishments of the U.S. Naval Oceanographic Office, the U.S. Naval Observatory, and Her Majesty's Nautical Almanac Office, Royal Greenwich Observatory. The tables in identical format are being published by the British Ministry of Defence as a Hydrographic Department publication.

Users should refer corrections, additions, and comments for improving this product to:

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# CONTENTS

# EXPLANATORY NOTE

The Introduction is divided into five sections as follows:

**A. Description of Tables.** This section contains brief notes on the purpose of the tables, their arrangement and a description of the Interpolation Table.

**B. Normal Usage.** For almost all purposes of marine navigation it suffices to use the tables in their simplest form; this section is therefore restricted to instructions, with illustrative examples, of that simple usage.

**C. Altitude Interpolation, Special Techniques.** Occasionally in marine navigation greater precision is required than that obtainable by the simple methods of section B; some other usages may require the full precision of the tables. In this section details are given of the methods that can be used to obtain this greater precision, with special reference to some pertinent solutions. A graphical means of interpolating altitude for latitude and local hour angle, i.e., making possible plotting from the DR or estimated position, is provided. Included also are methods for improving the accuracy of positioning using the usual position line and special polar plotting procedures.

**D. Other Applications.** The complete coverage of these tables makes them useful in the solution of many special navigational problems. Details are given in this section of some specific applications.

**E. Background.** This section relates to the accuracy of the tables and the reduction of sights, and some details are given concerning the production of the tables.

Most navigators need concern themselves only with sections A and B. Section C should, however, be referred to in unusual circumstances. Section D supplements and extends the usage to include some special applications and techniques.

# TERMINOLOGY

*Altitude*—angular distance above, or below, the horizon, positive or negative, respectively.

*Sextant Altitude*—the altitude reading taken from the sextant, modified by application of instrument error.

*Apparent Altitude* (App. Alt.)—the sextant altitude modified by application of index error, personal error and dip.

*Observed Altitude* (Ho)—the apparent altitude corrected as necessary for semidiameter, parallax, refraction, etc.

*Tabular Altitude* (Tab. Alt.)—the altitude tabulated in sight reduction tables for integral degrees of latitude, declination, and local hour angle.

*Calculated (or Computed) Altitude* (Hc)—tabular altitude interpolated for increments of the three pertinent arguments. Should no interpolation be required for any arguments, then the tabular altitude and the calculated altitude are identical; thus the columns of tabular altitudes for integral degree arguments of latitude, declination, and local hour angle are designated Hc.

*Computed Altitude* (Hc)—see Calculated Altitude.

*Altitude Difference* (d)—the tabular difference in minutes between successive tabular altitudes given in these tables.

*Second Differences*—the differences between successive values of d, not tabulated.

*Double-Second Difference* (DSD)—a quantity used in the more precise interpolation of altitude for declination. It is the algebraic difference between the tabular values of d immediately below and immediately above that on the line defined by the integral portion of the actual declination.

*Intercept*—(Hc-Ho), or (Ho-Hc); Hc greater than Ho, intercept is away (A); Ho greater than Hc, intercept is toward (T).

*Azimuth* (Zn)—the angle between the meridian and the vertical circle through the celestial body or point, measured eastward from north through 360°.

*Azimuth Angle* (Z)—the angle between the meridian and vertical circle through the celestial body or point, measured eastward or westward through 180° from north or south according to the elevated pole; the tabular values included in most sight reduction tables.

*Tabular Azimuth Angle* (Tab. Az., Z)—the azimuth angle as tabulated in these volumes for the three entering arguments.

*Tabular Declination* (Tab. Dec.)—the integral degree of declination used as an argument for entering these tables.

*Local Hour Angle* (LHA or L.H.A., *P*°)—the angle at the pole between the meridian of the observer and the meridian of the observed body, measured westward from the meridian of the observer through 360°; one of the three arguments for entering these tables.

*Meridian Angle*—the angular distance of a celestial body or point measured eastward or westward from the local meridian through 180°.

*Assumed (or Chosen) Latitude* (aL), *Assumed (or Chosen) Longitude* (aλ)—geographical coordinates assumed to facilitate entering these tables.

*Declination Increment* (Dec. Inc.)—the excess of the actual declination over the tabular declination; argument for entering Interpolation Table.

*Tabular Latitude* (Tab. Lat.)—latitude argument for entering these tables.

*Tabular Local Hour Angle* (Tab. LHA)—local hour angle argument for entering these tables.

*Latitude Increment*—difference between the actual latitude and that used as table argument.

*Local Hour Angle Increment*—difference between the actual local hour angle and that used as table argument.

*Argument*—one of the values used for entering a table.

# INTRODUCTION

## A. DESCRIPTION OF TABLES

**1. Purpose and Scope.** The main purpose of these tables is to facilitate the practice of astronomical navigation at sea. A secondary purpose is to provide, within the limits of the navigational precision required, a fundamental table of the solutions of a spherical triangle in which two sides and the included angle are given.

The tables have been designed primarily for use with the intercept or Marcq Saint Hilaire method of sight reduction, utilizing a position assumed or chosen, such that interpolation for latitude and local hour angle is not required.

Altitudes and azimuths are tabulated for all combinations of latitude, local hour angle and declination, at a uniform interval of 1° in each of the three arguments; but interpolation, to the highest precision, is provided only when interpolating altitude for declination. Within this scope, the aim has been to achieve absolute accuracy with a high standard of design and presentation.

**2. Arrangement.**

***(a) Format.*** The tables are divided into six volumes, each of which includes two eight-degree zones or sections of latitude. An overlap of 1° occurs between volumes, making 0°, 15°, 30°, 45°, 60°, 75° and 90° the first and/or last latitudes of the several volumes.

Within each of the 12 zones of 8° of latitude, for which the 182 pages of tabulations constitute a self-contained entity, the main argument is local hour angle, measured westward from the local meridian from 0° through 360°; to each integral degree of the local hour angle (LHA = $P$°) in the range 0° to 90° there corresponds an opening of two facing pages, which together contain the tabulations for:

(i) local hour angles, LHA = $P$° and 360° – $P$°, for latitude and declination of both same and contrary name;

(ii) local hour angles, LHA = 180° – $P$° and 180° + $P$°, for latitude and declination of same name.

These values of the local hour angle, which are the primary entering arguments, are prominently displayed at the top and bottom of each page; the horizontal argument heading each column is latitude, and the vertical argument is declination.

Within each opening, the left-hand page is always restricted to the tabulations for local hour angles LHA = $P$° and 360° – $P$°, and for declinations of the same name. The right-hand page contains: on the upper portion, the tabulations for local hour angles LHA = $P$° and 360° – $P$°, and declinations of contrary name; and, on the lower portion, the tabulations for supplementary local hour angles LHA = 180° – $P$° and 180° + $P$°, and declinations of the same name.

The two portions are separated, in each column, by a horizontal rule which, together with the vertical lines, form a configuration across the page resembling the profile of a staircase. Hereafter this line separating data of Contrary and Same name will be referred to as the C-S Line. The horizontal segments of this line indicate the degree of declination in which the horizon occurs. The tabulated altitudes are continuous across this horizontal rule, provided that those on one side are regarded as being negative, that is depressions below the horizon. Interpolation within this one-degree interval of declination is described in section C.2.

With this arrangement, the opening required for the reduction of a sight taken within a particular zone of latitude, either north or south, is determined uniquely by the value of the local hour angle. Whatever the local hour angle, tabulations for that entry and for all declinations and all latitudes within the zone are immediately available at the one opening.

***(b) Tabulated Quantities.*** For each combination of arguments there is tabulated: the altitude Hc to 0.1′; in smaller type, the actual difference d of the tabular altitude from that declination entry to that for the next higher degree, referred to later as the altitude difference, together with its sign; the azimuth angle, Z, to 0.1°.

Rules are given at each opening for converting the tabular azimuth angle into true azimuth, measured eastward from north; the rules differ according to the hemisphere—northern or southern—and the range of local hour angle.

***(c) Interpolation Table.*** Provision is made for interpolation of the tabular altitude for declination by the tabulation of the altitude difference, d, and by the Interpolation Table included in each volume. The Interpolation Table, abbreviated Int. Tab. in examples, is designed to make possible a reasonably high degree of precision in interpolating the altitude to the nearest 0.1′ of declination.

The main, vertical, argument of the Interpolation Table is the excess of the actual declination over that used (an integral degree) as the tabular entry. Since it is recommended that the tabular entry used should always be the integral degree of declination numerically less than or equal to the actual declination, this excess should always be the actual minutes of the declination referred to as the declination increment, abbreviated Dec. Inc. The other argument is the tabular altitude difference, d, which for convenience is divided into two parts, the first being a multiple of 10′ (10′, 20′, 30′, 40′, or 50′) and the second the remainder in the range 0.0′ to 9.9′. The minutes of this remainder appear as the horizontal argument, the decimal part as the vertical argument, in the subtable, which is given opposite each range of one minute (10 entries) of the Dec. Inc. used in entering the tables.

The right-hand column of each vertically divided half-page of the Interpolation Table contains a series of critical tables, each corresponding to the range of Dec. Inc. opposite which it is placed, which give the correction for the effect. of second differences.

The Interpolation Table was designed so that it could be printed on four pages. The inside front cover and facing page provide for the range 0.0′ to 31.9′ of the Dec. Inc., while the inside back cover and facing page provide for the range 28.0′ to 59.9′. No special table is provided for interpolation of the azimuth angle and the differences are not tabulated.

## B. NORMAL USAGE

**1. The Intercept Method.** The standard practice at sea is to draw the position line, corresponding to the observed altitude of a heavenly body, as the perpendicular to the intercept (difference between observed and calculated altitude) plotted toward or away from the observed body, using a position so chosen as to facilitate the calculation of the altitude and azimuth. As with the *Tables of Computed Altitude and Azimuth*, H.O. Pub. No. 214—H.D. 486 in the United Kingdom—the normal function of these tables is to provide the calculated altitude and azimuth, interpolated to the declination of the observed body, for such an assumed position; interpolation for latitude and local hour angle is not required. The procedures of choosing an assumed position, and of plotting the resulting position line, are so familiar that they will not be described.

**2. Entering Arguments.** The entering arguments, obtained from a knowledge of position, the observed body, and the time of observation, utilizing standard procedures and data from *The Nautical Almanac*, are:

(*a*) the integral degree of latitude nearest to the estimated latitude of the observer (abbreviated Tab. Lat.);

(*b*) the integral degree of local hour angle obtained by applying to the GHA of the observed body a longitude nearest to the estimated longitude of the observer (abbreviated Tab. LHA);

(*c*) the declination of the observed body (abbreviated Dec.), labeled SAME or CONTRARY as the case may be; it is expressed as integral degrees (referred to as Tab. Dec.) together with minutes and decimal parts thereof, the minutes and decimal parts being referred to as the declination increment (Dec. Inc.).

**3. Instructions for Use.** Normally (probably for 99 percent of all observations) the second-difference correction in the interpolation of the altitude for declination can, and should, be ignored. It is, however, convenient to describe the application of the correction at the same time as the main part of the interpolation; the additional instructions pertaining to this correction are indicated by an asterisk (*) and can be ignored for normal usage. A further discussion of interpolation utilizing second differences is given in section C. 1.

The following instructions for use of the tables and the extracted tabular data are given in precise detail; they will be greatly simplified in practice.

*(a)* Choose the volume and section corresponding to the zone in which the Lat. is contained.

*(b)* In this section select the opening according to the LHA, then choose the page according to the following rules:

| | |
|---|---|
| LHA 0° to 90°, 360° to 270°, | SAME NAME, enter left-hand page.<br>CONTRARY NAME, enter right-hand page (top). |
| LHA 180° to 90°, 180° to 270°, | SAME NAME, enter right-hand page (bottom). |

*(c)* For the LHA and the pertinent page, select the column headed Lat. and the line corresponding to Tab. Dec.

*(d)* Record:

(i) the tabular altitude, Hc;

(ii) the altitude difference, d;

*(iii) if high precision is required, or if d is printed in italic type followed by a dot, form the double-second difference by determining mentally the difference between the tabular values of d immediately below and immediately above the d which corresponds to the integral portion of the actual declination; the difference is always negative, but the sign need not be recorded.

(iv) the tabular azimuth angle, Z, interpolated mentally to the actual declination (Dec.).

*(e)* Convert the interpolated azimuth angle, Z, to the true azimuth, Zn, measured eastward from north, according to the rules to be found at each opening of the tables.

*(f)* Turn to the Interpolation Table, on the inside front cover and facing page if the Dec. Inc. is in the range 0.0′ to 31.9′, or on the inside back cover and facing page if the Dec. Inc. is in the range 28.0′ to 59.9′.

*(g)* Extract the two tabular parts of the interpolation correction corresponding to the Dec. Inc.:

(i) the first part: the tabular value corresponding to the Dec. Inc. and the tens of minutes of the altitude difference, d;

(ii) the second part: the tabular value corresponding to the Dec. Inc. and the remainder of d; this is to be taken from the subtable opposite (on the right-hand side of) the range of 1′ of the vertical argument in which the Dec. Inc. occurs; in this subtable the units of d form the horizontal argument and the decimals the vertical argument.

*(h)* Add the two parts to form the interpolation correction, and apply this to the tabular altitude, with the sign of the altitude difference, to give the altitude interpolated for declination, i.e., the calculated altitude.

**(i)* If precision is required, and if the double-second difference has been recorded, take out the second-difference correction (corr.) from the small critical table, with argument Double Second Diff. opposite the same line of the Dec. Inc. in the Interpolation Table.

**(j)* Add the second-difference correction to the altitude, interpolated for declination as in *(h)*, to obtain a more precise altitude.

**4. Illustrations of Use.**

*(a)* Primarily as an illustration of the use of the Interpolation Table, the calculated altitude and true azimuth are determined for Lat. 76° S, LHA 29°, and Dec. 69°34.8′ S. Data are exhibited in Fig. 1.

**29°, 331° L.H.A.**

**LATITUDE SAME NAME**

| Dec. | 75° Hc | 75° d | 75° Z | 76° Hc | 76° d | 76° Z | 77° Hc | 77° d | 77° Z |
|---|---|---|---|---|---|---|---|---|---|
| ° | ° ′ | ′ | ° | ° ′ | ′ | ° | ° ′ | ′ | ° |
| **65** | 76 11.4 | + 50.4 | 120.9 | 75 39.1 | + 52.4 | 124.2 | 75 04.0 | + 54.0 | 127.3 |
| **66** | 77 01.8 | + 49.0 | 118.5 | 76 31.5 | + 51.2 | 122.2 | 75 58.0 | + 53.1 | 125.6 |
| **67** | 77 50.8 | + 47.3 | 115.9 | 77 22.7 | + 49.9 | 119.9 | 76 51.1 | + 52.1 | 123.6 |
| **68** | 78 38.1 | + *45.1* | •112.8 | 78 12.6 | + 48.3 | 117.3 | 77 43.2 | + 50.9 | 121.4 |
| **69** | 79 23.2 | + *42.5* | •109.4 | 79 00.9 | + *46.2* | •114.3 | 78 34.1 | + 49.2 | 118.8 |
| **70** | 80 05.7 | + *39.4* | •105.4 | 79 47.1 | + *43.6* | •110.8 | 79 23.3 | + *47.3* | •115.8 |
| **71** | 80 45.1 | + *35.4* | •100.9 | 80 30.7 | + *40.5* | •106.8 | 80 10.6 | + *44.8* | •112.3 |
| **72** | 81 20.5 | + *30.6* | • 95.7 | 81 11.2 | + *36.4* | •102.1 | 80 55.4 | + *41.7* | •108.3 |
| **73** | 81 51.1 | + *24.8* | • 89.8 | 81 47.6 | + *31.6* | • 96.7 | 81 37.1 | + *37.6* | •103.5 |
| **74** | 82 15.9 | + *18.2* | • 83.2 | 82 19.2 | + *25.6* | • 90.6 | 82 14.7 | + *32.7* | • 98.0 |
| **75** | 82 34.1 | + *10.7* | • 76.0 | 82 44.8 | + *18.5* | • 83.6 | 82 47.4 | + *26.3* | • 91.5 |

Data from page 60

**INTERPOLATION TABLE**

| Dec. Inc. | Tens 10′ | 20′ | 30′ | 40′ | 50′ | Decimals | Units 0′ | 1′ | 2′ | 3′ | 4′ | 5′ | 6′ | 7′ | 8′ | 9′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ | ′ |
| **34.0** | 5.6 | 11.3 | 17.0 | 22.6 | 28.3 | .0 | 0.0 | 0.6 | 1.1 | 1.7 | 2.3 | 2.9 | 3.4 | 4.0 | 4.6 | 5.2 |
| **34.1** | 5.7 | 11.3 | 17.0 | 22.7 | 28.4 | .1 | 0.1 | 0.6 | 1.2 | 1.8 | 2.4 | 2.9 | 3.5 | 4.1 | 4.7 | 5.2 |
| **34.2** | 5.7 | 11.4 | 17.1 | 22.8 | 28.[illegible] | .2 | 0.1 | 0.7 | 1.3 | 1.8 | 2.4 | 3.0 | 3.6 | 4.1 | 4.7 | 5.3 |
| **34.3** | 5.7 | 11.4 | 17.1 | 22.9 | 28.6 | .3 | 0.2 | 0.7 | 1.3 | 1.9 | 2.5 | 3.0 | 3.6 | 4.2 | 4.8 | 5.3 |
| **34.4** | 5.7 | 11.5 | 17.2 | 22.9 | 28.7 | .4 | 0.2 | 0.8 | 1.4 | 2.0 | 2.5 | 3.1 | 3.7 | 4.3 | 4.8 | 5.4 |
| **34.5** | 5.8 | 11.5 | 17.3 | 23.0 | 28.8 | .5 | 0.3 | 0.9 | 1.4 | 2.0 | 2.6 | 3.2 | 3.7 | 4.3 | 4.9 | 5.5 |
| **34.6** | 5.8 | 11.5 | 17.3 | 23.1 | 28.8 | .6 | 0.3 | 0.9 | 1.5 | 2.1 | 2.6 | 3.2 | 3.8 | 4.4 | 4.9 | 5.5 |
| **34.7** | 5.8 | 11.6 | 17.4 | 23.2 | 28.9 | .7 | 0.4 | 1.0 | 1.6 | 2.1 | 2.7 | 3.3 | 3.9 | 4.4 | 5.0 | 5.6 |
| **34.8** | 5.8 | 11.6 | 17.4 | 23.2 | 29.0 | .8 | 0.5 | 1.0 | 1.6 | 2.2 | 2.8 | 3.3 | 3.9 | 4.5 | 5.1 | 5.6 |
| **34.9** | 5.9 | 11.7 | 17.5 | 23.3 | 29.1 | .9 | 0.5 | 1.1 | 1.7 | 2.2 | 2.8 | 3.4 | 4.0 | 4.5 | 5.1 | 5.7 |
| **35.0** | 5.8 | 11.6 | 17.5 | 23.3 | 29.1 | .0 | 0.0 | 0.6 | 1.2 | 1.8 | 2.4 | 3.0 | 3.5 | 4.1 | 4.7 | 5.3 |

| Double Second Diff. ′ | and Corr. ′ |
|---|---|
| 0.8 | |
| | 0.1 |
| 2.5 | |
| | 0.2 |
| 4.1 | |
| | 0.3 |
| 5.8 | |
| | 0.4 |
| 7.4 | |
| | 0.5 |
| 9.1 | |
| | 0.6 |
| 10.7 | |
| | 0.7 |
| 12.3 | |
| | 0.8 |
| 14.0 | |
| | 0.9 |
| 15.6 | |
| | 1.0 |
| 17.3 | |
| | 1.1 |
| 18.9 | |
| | 1.2 |
| 20.6 | |

Data from Interpolation Table

FIGURE 1

The latitude zone required is contained in the first half (covering latitudes 75° to 82°) of Volume **6**; the tabular entries are found on the left-hand page of the opening for LHA 29° (top of page), in the column for Lat. 76°, on the line for Dec. 69°, i.e., for latitude and declination of the SAME name. The data taken from page 60 and recorded are:

| | | |
|---|---|---|
| tabular altitude, Hc | | 79°00.9′ |
| altitude difference, d | | (+)46.2 |
| * double-second difference | (43.6′ – 48.3′) | 4.7 |
| interpolated azimuth angle, Z | (114.3° – 2.0°) | 112.3° |
| true azimuth, Zn | (180° + 112.3°) | 292.3° |

Note that the Dec. Inc. 34.8′ is the main argument for entering the Interpolation Table to extract the first correction for tens of minutes of altitude difference, d, and that it also indicates the subtable where the second part of the correction for units and decimal parts (minutes) of altitude difference, d, is found.

| | | Dec. Inc. | Alt. Diff., d | Correction |
|---|---|---|---|---|
| Interpolation Table | 1st part | 34. 8′ | 40′ | 23.2′ |
| | 2nd part | 34. 8′ | 6.2′ | 3.6′ |
| First-diff. correction | | 34. 8′ | 46.2′ | 26.8′ |
| *Second-diff. correction | | 34. 8′ | DSD= 4.7′ | 0.3′ |

The interpolated altitude, using first differences only, is thus 79°00.9′ + 26.8′ = 79°27.7′, since d is positive; the correction for second differences makes this 79°28.0′, since the correction is always positive.

The Interpolation Table should always be entered with the arguments as arranged, i.e., with Dec. Inc. as vertical argument and with d as horizontal argument.

No special table is provided for interpolation of the azimuth angle, and the differences are not tabulated. The successive azimuth differences are less than 10.0° for altitudes less than 84°, and can easily be found by inspection; to this point second differences can be neglected. If formal interpolation for azimuth is necessary, treat the degrees as if minutes and obtain the correction from the unit-decimal portion of the Interpolation Table; but for most practical applications interpolation by inspection will probably suffice. In this illustration formal interpolation, using an azimuth difference of –3.5° and the right-hand block of the Interpolation Table corresponding to the Dec. Inc. of 34.8′, gives a correction of –2.0°.

*(b) Required.*—The altitude and azimuth for the following arguments:

| | | ***Example i*** | | | ***Example ii*** | | |
|---|---|---|---|---|---|---|---|
| LHA | | 22° | | | 22° | | |
| Lat. | | 75° N | | | 77° S | | |
| Dec. | | 5°14.3′ S | | | 2°27.6′ N | | |
| Name | | Contrary | | | Contrary | | |
| | | Hc | d | Z | Hc | d | Z |
| From page 47 | | 8°54.6′ | (–)59.7′ | 157.8°† | 10°02.8′ | (–)59.8′ | 157.7°† |
| Int. Tab. | 1st part | (–)11.9 | | | (–)23.0 | | |
| | 2nd part | (–) 2.3 | | | (–) 4.5 | | |
| Hc & Zn | | 8°40.4′ | | 202.2° | 9°35.3′ | | 337.7° |

| | | ***Example iii*** | | | ***Example iv*** | | |
|---|---|---|---|---|---|---|---|
| LHA | | 202° | | | 202° | | |
| Lat. | | 79° N | | | 81° S | | |
| Dec. | | 65°43.2′ N | | | 74°50.7′ S | | |
| Name | | Same | | | Same | | |
| | | Hc | d | Z | Hc | d | Z |
| From page 47 | | 54°34.6′ | (+)59.5′ | 15.7°† | 65°25.7′ | (+)59.4′ | 14.1°† |
| Int. Tab. | 1st part | (+)36.0 | | | (+)42.3 | | |
| | 2nd part | (+) 6.9 | | | (+) 7.9 | | |
| Hc & Zn | | 55°17.5′ | | 015.7° | 66°15.9′ | | 165.9° |

†The values of Z are interpolated mentally where necessary.

*(c) Required.*—The solution and fix for the following simultaneous observations made from DR position 77°43′ N, 10°12′ W, at GMT $04^{h}48^{m}17^{s}$; the observed altitudes were: *Regulus* 13°10.1′, *Vega* 28°00.3′, and *Hamal* 32°56.1′. From *The Nautical Almanac*:

| | *Regulus* | | *Vega* | | *Hamal* | |
|---|---|---|---|---|---|---|
| GHA♈ $04^{h}$ | 66°00.6′ | | 66°00.6′ | | 66°00.6′ | |
| Inc. $48^{m}17^{s}$ | 12°06.2′ | | 12°06.2′ | | 12°06.2′ | |
| GHA♈ $04^{h}48^{m}17^{s}$ | 78°06.8′ | | 78°06.8′ | | 78°06.8′ | |
| SHA & Dec. | 208°19.6′ | 12°07.4′ N | 81°01.9′ | 38°45.4′ N | 328°38.7′ | 23°19.0′ N |
| GHA ☆ | 286°26.4′ | | 159°08.7′ | | 46°45.5′ | |
| Assumed Long. (aλ) | 10°26.4′ W | | 10°08.7′ W | | 9°45.5′ W | |
| LHA | 276°00.0′ | | 149°00.0′ | | 37°00.0′ | |

Enter Tables in Lat. 78° with LHA and Dec. as above.

| | | Hc | d | Z | Hc | d | Z | Hc | d | Z |
|---|---|---|---|---|---|---|---|---|---|---|
| Pages 170, 65, 76 | | 12°58.9′ | (+)58.6′ | 93.3°† | 27°30.1′ | (+)59.5′ | 27.1°† | 32°20.8′ | (+)59.3′ | 138.9°† |
| Int. Tab. | 1st part | (+) 6.2 | | | (+)37.8 | | | (+)15.8 | | |
| | 2nd part | (+) 1.1 | | | (+) 7.2 | | | (+) 3.0 | | |
| Hc | | 13°06.2′ | | | 28°15.1′ | | | 32°39.6′ | | |
| Ho | | 13°10.1′ | | | 28°00.3′ | | | 32°56.1′ | | |
| Intercept & Zn | | 3.9 T | | 093.3° | 14.8 A | | 332.9° | 16.5 T | | 221.1° |

†Interpolated value.

The plot is shown in Figure 2.

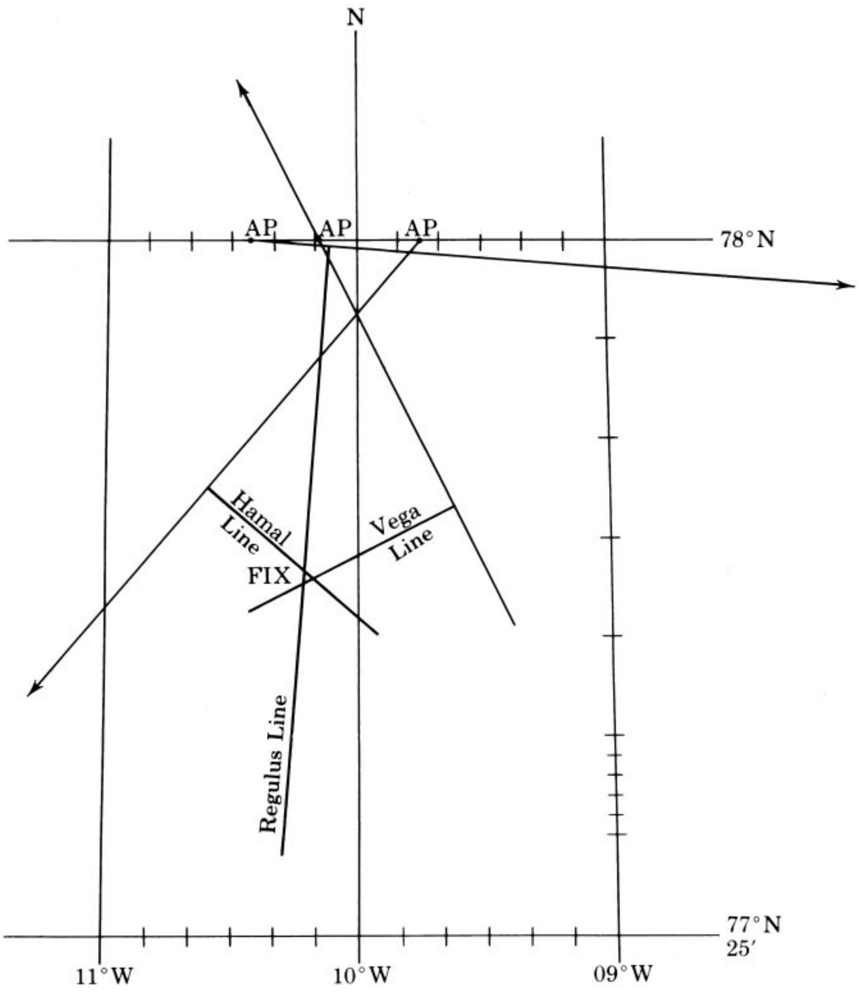



**FIGURE 2**

## C. ALTITUDE INTERPOLATION, SPECIAL TECHNIQUES

**1. Interpolation when Second Differences are Required.** In section B instructions were given for the interpolation of altitude for increments of declination, and it was emphasized that normally it will not be required to include the effect of second differences in the interpolation. An illustration was included, however, exhibiting how the second-difference correction was applied when necessary. Usually the precision of the interpolation in these tables using only first differences decreases as the altitude increases. For altitudes very near the zenith, where even the inclusion of the correction for second differences is inadequate, other techniques can be employed (see section C.4). When the altitude difference, d, is printed in italic type, additionally accompanied by a small dot alongside, the second-difference correction may exceed 0.25′, and should normally be applied; if a precision better than this is required, the second-difference correction may be applied even when the altitude difference, d, is not in italic type.

To facilitate interpolation using second differences a critical table (see section A.2) is included as the last column of each half-page of the Interpolation Table, giving the correction when entered with the argument double-second difference, DSD. The DSD required for entering this critical table is, as stated in section B.3, the difference between the tabular values of d immediately below and immediately above the d which corresponds to the integral portion of the actual declination. When the tabular DSD (critical) entry corresponds to an exact tabular value always use the upper of the two possible corrections, for example with a Dec. Inc. entry between 8.0′ and 9.4′ and a DSD entry of 24.1′ the correction is 0.7′. Always use the compartment of the DSD table opposite the block of the Interpolation Table wherein was found the Dec. Inc. entry. The DSD correction is always added to the altitude already interpolated using first differences.

The following examples illustrate the interpolation of altitudes for declination when it is necessary to include the correction for second differences.

*Required*.—The altitude and azimuth for the following arguments:

| | | *Example i* | | | | *Example ii* | | |
|---|---|---|---|---|---|---|---|---|
| LHA | 22° | | | | 338° | | | |
| Lat. | 75° N | | | | 77° S | | | |
| Dec. | 69°36.4′ N | | | | 74°10.8′ S | | | |
| Name | | Same | | | | Same | | |
| | Hc | d | DSD | Z | Hc | d | DSD | Z |
| From page 46 | 81°01.8′ | (+)45.5′ | 5.9′ | 117.9°† | 83°46.8′ | (+)34.5′ | 13.6′ | 106.2°† |
| Int. Tab. 1st part | (+)24.3 | | | | (+) 5.4 | | | |
| Int. Tab. 2nd part | (+) 3.3 | | | | (+) 0.8 | | | |
| DSD Correction | (+) 0.3 | | | | (+) 0.5 | | | |
| Hc & Zn | 81°29.7′ | | | 242.1° | 83°53.5′ | | | 073.8° |

| | | *Example iii* | | | | *Example iv* | | |
|---|---|---|---|---|---|---|---|---|
| LHA | 22° | | | | 338° | | | |
| Lat. | 79° S | | | | 82° N | | | |
| Dec. | 85°47.1′ S | | | | 86°52.2′ N | | | |
| Name | | Same | | | | Same | | |
| Int. Tab. | Hc | d | DSD | Z | Hc | d | DSD | Z |
| From page 46 | 83°22.2′ | (–)48.7′ | 6.0′ | 12.6°† | 85°27.4′ | (–)47.6′ | 9.5′ | 13.1°† |
| Int. Tab. 1st part | (–)31.4 | | | | (–)34.8 | | | |
| Int. Tab. 2nd part | (–) 6.9 | | | | (–) 6.6 | | | |
| DSD Correction | (+) 0.3 | | | | (+) 0.3 | | | |
| Hc & Zn | 82°44.2′ | | | 192.6° | 84°46.3′ | | | 013.1° |

*Note*.—The second-difference correction is always positive.
†Azimuth angles are interpolated mentally.

**2. Interpolation near the Horizon.** This discussion is restricted to the interpolation of altitude for declination within the 1° interval containing the horizon, indicated by the horizontal segments of the C-S Line. Interpolation of altitude in the interval under consideration is accomplished by using the last tabular altitude and altitude difference appearing above the C-S Line, i.e., for this one degree of declination interval, interpolation is carried out across the C-S Line. Since the last tabular altitude above the C-S Line indicates the body's altitude above the horizon for LHA at top of page, for the pertinent latitude, and for the last integral declination above the horizontal segment of the C-S Line pertaining to that particular latitude, interpolation resulting in positive altitudes may be carried out for increments of declination of contrary name so long as the interpolated altitude correction does not exceed the last tabular altitude above the C-S Line; for the LHA at bottom of page, positive altitudes will result when interpolating altitude for increments of declination of same name so long as the interpolated altitude correction exceeds the last tabular value above the C-S Line. Interpolation for declinations and increments of declination in excess of the above limits result in negative altitudes or angles of depression—see section C.3.

The tabular azimuth angle pertinent to this one degree interval of declination is that immediately above or that immediately below the C-S Line, according as the entering arguments are contrary or same name, respectively. The difference in azimuth for the interval is determined by taking the value of tabular azimuth angle, on the same side of the C-S Line as the LHA argument, from the supplement of that on the opposite side of the line.

The following four examples illustrate variations in data and results obtained in the altitude interpolation procedure near the horizon.

| | | ***Example i*** | | | ***Example i*** | | |
|---|---|---|---|---|---|---|---|
| LHA | | 22° | | | 338° | | |
| Lat. | | 75° N | | | 78° S | | |
| Dec. | | 13°16.3′ S | | | 11°20.5′ N | | |
| | | Hc | d | Z | Hc | d | Z |
| From page 47 | | (+) 0°56.9′ | (–)59.8′ | 158.6°† | (+) 0°08.9′ | (–)59.8′ | 158.4°† |
| Int. Tab. | 1st part | (–) 13.6 | | | (–) 17.1 | | |
| | 2nd part | (–) 2.7 | | | (–) 3.3 | | |
| Total Correction | | (–) 16.3 | | | (–) 20.4 | | |
| Azimuth Angle, Z | | | | 158.6° | | | 158.4° |
| Hc & Zn | | (+) 0°40.6′ | | 201.4° | (–) 0°11.5′ | | 021.6° |

| | | ***Example iii*** | | | ***Example iv*** | | |
|---|---|---|---|---|---|---|---|
| LHA | | 158° | | | 158° | | |
| Lat. | | 80° N | | | 82° S | | |
| Dec. | | 9°24.6′ N | | | 7°10.3′ S | | |
| | | Hc | d | Z | Hc | d | Z |
| From page 47 | | (–) 0°17.1′‡ | (+)59.9′‡ | 158.3°† | (–) 0°25.4′‡ | (+)59.9′‡ | 158.2°† |
| Int. Tab. | 1st part | (+) 20.5 | | | (+) 8.6 | | |
| | 2nd part | (+) 4.0 | | | (+) 1.7 | | |
| Total Correction | | (+) 24.5 | | | (+) 10.3 | | |
| Azimuth Angle, Z | | | | 21.7°* | | | 21.8°* |
| Hc & Zn | | (+) 0°07.4′ | | 338.3° | (–) 0°15.1′ | | 201.8° |

* Supplements of the tabular azimuth angles.
† Interpolated value.
‡ When entering the tables with LHA at the bottom of the page reverse the signs of the values extracted from above the C-S Line.

In example i, for latitude and declination of contrary name and LHA at top of page, the altitude correction is numerically less than the last tabular altitude above the C-S Line, so that the resulting interpolated altitude is positive. In example ii, for latitude and declination of contrary name, and LHA at top of page, the correction is numerically greater than the tabular altitude, so that the resulting altitude is negative. In example iii, for latitude and declination of same name, and LHA at bottom of page, the correction is greater than the tabular altitude, which is negative, so that the resulting altitude is positive. In example iv, for latitude and declination of same name and LHA at bottom of page, the correction is less than the tabular altitude, so that the resulting altitude is negative.

**3. Negative Altitudes.** This paragraph is restricted to tabular and interpolated altitudes for declinations other than one-degree intervals of declination containing the C-S Line. The latter is fully discussed in section C.2. Although negative altitudes are not tabulated as such within these volumes, they are immediately available to meet any requirement. For instance, for all local hour angles at the top of the right-hand page, all tabular or interpolated altitudes on that page for declination below the C-S Line, are negative; also for any local hour angle at the bottom of the right-hand page, all tabular or interpolated altitudes for declinations above the C-S Line, are negative; additionally, for these same local hour angles the tabular or interpolated altitudes for the left-hand page are negative. Interpolation of altitude for declination increments within these areas of negative altitude should, however, be accomplished as if the altitudes were positive, adhering strictly to the sign given d. Then, after interpolation, regard the results as negative. In all instances involving negative altitudes, except the one-degree interval of declination which includes the C-S Line, the supplement of the pertinent tabular azimuth angle is that to be converted to true azimuth by the rules to be found on each opening of the basic tables.

**4. Interpolation near the Zenith**. The altitude near the zenith is difficult to interpolate by the usual methods and the azimuth angle is indeterminate at the zenith.

For altitudes less than 86°—i.e., for zenith distances greater than 4°—interpolation of the tabular altitude for declination, utilizing both first and second differences and the Interpolation Table, may be made to within about 0.2′; linear interpolation for azimuth angle can be made to about 0.2°. Closer to the zenith, not only do second differences exceed the limits of the tables but higher differences are also significant. Within this region of a few degrees from the zenith, where normal interpolation methods are inadequate, the following method of using the tables for plotting a position line is recommended.

The method is based on the fact that usually both altitude and azimuth can be readily interpolated when the difference in the entering arguments, latitude ~ declination, remains constant. The position line may thus be plotted with the intercept determined by the altitude interpolated for equal intervals of latitude and declination, which is in this instance plotted from the pertinent integral latitude, increased by the increment of declination for which interpolation was made. The Interpolation Table is employed in carrying out the desired interpolation, but it should be noted that in this instance the values of altitude and azimuth extracted from the basic tables constitute data which require independent differencing; the tabular altitude difference, d, is not used.

To carry out the altitude interpolation, the basic tables are entered with the pertinent LHA and Dec. and, with the integral degree of Lat. so chosen that when increased by the declination increment, it is within 30′ of the known or DR latitude; this practice will prevent long intercepts. For these entering arguments and for a latitude and declination one degree less and one degree more than the above referenced latitude and declination, respectively, extract the tabular altitudes and azimuths. The altitudes and azimuths are then differenced and with these differences interpolation of altitude and azimuth for the desired declination is made, utilizing the Interpolation Table. The calculated altitude is then compared with that observed to determine the intercept, which together with the interpolated azimuth makes possible the construction of a position line, which is plotted from the assumed longitude, and from the latitude of the entering argument, augmented by the declination increment.

The following examples illustrate the preceding method utilizing the tabulations from page 12. Note that altitude and azimuth angle are interpolated to the required declination using the Interpolation Table but with arguments Dec. Inc. and altitude differences formed.

| Example | LHA | Lat. | Dec. | Ho |
|---|---|---|---|---|
| i | 5° 18′ | 76° 15′ S | 75° 43.2′ S | 88° 36.6′ |
| ii | 355 22 | 78 43 N | 77 06.4 N | 88 07.2 |
| iii | 4 46 | 80 24 N | 79 52.1 N | 89 01.5 |
| iv | 355 03 | 82 09 S | 80 58.5 S | 88 37.2 |

From page 12 (LHA = 5° or 355°, latitude and declination Same Name)

***Example i***

| Lat. | Dec. | Tab. Hc | diff. | Tab. Z | diff. |
|---|---|---|---|---|---|
| 75° | 74° | 88°19.9′ | | 124.4° | |
| | | | (+) 4.0′ | | (+) 1.8° |
| 76° | 75° | 88°23.9′ | | 126.2° | |
| | | | (+) 3.9′ | | (+) 1.9° |
| 77° | 76° | 88°27.8′ | | 128.1° | |

Interpolate to Dec.=75° 43.2′
Dec. Inc. = 43.2′, diff. = (+)3.9′ and Z diff. = (+)1.9°

| | | | | |
|---|---|---|---|---|
| Tab. Hc | | 88°23.9′ | Tab. Z | 126.2° |
| Int. Tab. | 1st part | 0.0′ | | |
| | 2nd part | (+) 2.8′ | | (+) 1.4′ |
| | Hc | 88°26.7′ | Z | 127.6° |
| | Ho | 88°36.6′ | | |
| Intercept | | 9.9′ T | Zn | 307.6° |
| | Plot from Lat. 76°43.2′ S | | | |

***Example ii***

| Lat. | Dec. | Tab. Hc | diff. | Tab. Z | diff. |
|---|---|---|---|---|---|
| 78° | 76° | 87°42.4′ | | 148.2° | |
| | | | (+) 2.5′ | | (+) 1.9° |
| 79° | 77° | 87°44.9′ | | 150.1° | |
| | | | (+) 2.3′ | | (+) 1.9° |
| 80° | 78° | 87°47.2′ | | 152.0° | |

Interpolate to Dec.=77° 06.4′
Dec. Inc. = 06.4′, diff. = (+)2.3′ and Z diff. = (+)1.9°

| | | | | |
|---|---|---|---|---|
| Tab. Hc | | 87°44.9′ | Tab. Z | 150.1° |
| Int. Tab. | 1st part | 0.0′ | | |
| | 2nd part | (+) 0.2′ | | (+) 0.2′ |
| | Hc | 87°45.1′ | Z | 150.3° |
| | Ho | 88°07.2′ | | |
| Intercept | | 22.1′ T | Zn | 150.3° |
| | Plot from Lat. 79°06.4′ S | | | |

***Example iii***

| Lat. | Dec. | Tab. Hc | diff. | Tab. Z | diff. |
|---|---|---|---|---|---|
| 79° | 78° | 88°35.3′ | | 132.6° | |
| | | | (+) 3.6′ | | (+) 2.6° |
| 80° | 79° | 88°38.9′ | | 135.2° | |
| | | | (+) 3.4′ | | (+) 2.8° |
| 81° | 80° | 88°42.3′ | | 138.0° | |

Interpolate to Dec.=79° 52.1′
Dec. Inc. = 52.1′, diff. = (+)3.4′ and Z diff. = (+)2.8°

| | | | | |
|---|---|---|---|---|
| Tab. Hc | | 88°38.9′ | Tab. Z | 135.2° |
| Int. Tab. | 1st part | 0.0′ | | |
| | 2nd part | (+) 3.0′ | | (+) 2.4′ |
| | Hc | 88°41.9′ | Z | 137.6° |
| | Ho | 89°01.5′ | | |
| Intercept | | 19.6′ T | Zn | 222.4° |
| | Plot from Lat. 80°52.1′ S | | | |

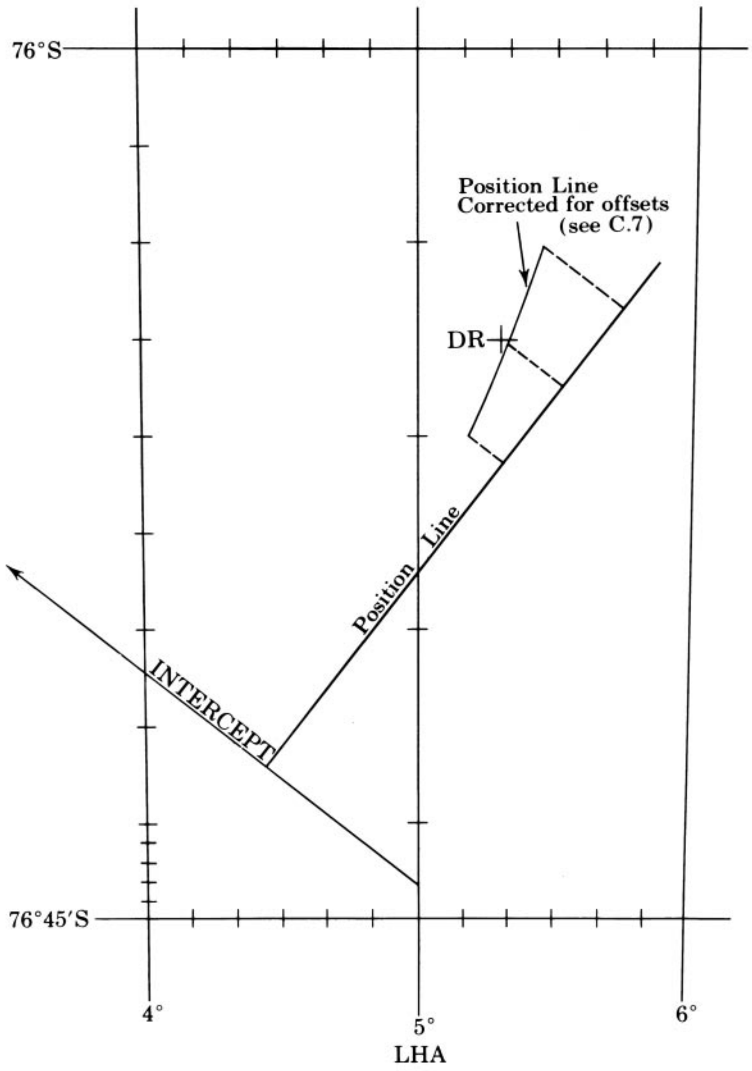
***Example iv***

| Lat. | Dec. | Tab. Hc | diff. | Tab. Z | diff. |
|---|---|---|---|---|---|
| 80° | 79° | 88°38.9′ | | 135.2° | |
| | | | (+) 3.4′ | | (+) 2.8° |
| 81° | 80° | 88°42.3′ | | 138.0° | |
| | | | (+) 3.1′ | | (+) 3.0° |
| 82° | 81° | 88°45.4′ | | 141.0° | |

Interpolate to Dec.=80° 58.5′
Dec. Inc. = 58.5′, diff. = (+)3.1′ and Z diff. = (+)3.0°

| | | | | |
|---|---|---|---|---|
| Tab. Hc | | 88°42.3′ | Tab. Z | 138.0° |
| Int. Tab. | 1st part | 0.0′ | | |
| | 2nd part | (+) 3.0′ | | (+) 2.9′ |
| | Hc | 88°45.3′ | Z | 140.9° |
| | Ho | 88°37.2′ | | |
| Intercept | | 8.1′ A | Zn | 039.1° |
| | Plot from Lat. 81°58.5′ S | | | |

A plot of Example i is shown in Figure 3.




Plot of Example (i); the Position Line has been corrected using offsets as described in Section C.4 and Section C.7.

**FIGURE 3**

**5. Indeterminate Azimuths.** When the body is in the zenith its azimuth is indeterminate, that is when LHA = 0° and when latitude and declination are equal and have the Same Name. In these cases Z is tabulated as 90° or as one-half the preceding value. There are 91 of these cases.

When latitude = 90° and declination = 90° the altitude = 90° for all hour angles. Here the value of Z tabulated is one-half the preceding value. There are 182 of these cases, two of which are included in the previous set. In the above cases the tabulated azimuth angles are the mathematical limits of the azimuth angle when the limit is approached in a specified direction.

In the special cases when the latitude = 90°, i.e., at the poles, all directions from the North Pole are south and from the South Pole are north; the criterion adopted in these cases has been to tabulate the azimuth as equal to 180° minus LHA, i.e., the directions are tabulated as the angular directions from the lower branch of the Greenwich meridian. There are 90 × 180 of these cases not included in the previous sets.

**6. Interpolation for Latitude and Local Hour Angle.** Although interpolation of altitude will normally be required for declination only, there will be instances in which interpolation of altitude for latitude and local hour angle will also be required. At a tabular interval of 1°, interpolation of the altitude for latitude or local hour angle is not always linear; numerical interpolation to the full precision of the tables would be quite impracticable, even if differences or other aids to interpolation were provided. The difficulty can, however, be avoided by the use of the following method.

In principle the method consists of constructing, on a constant-scale auxiliary plotting sheet (Diagram C), the intercept between the zenith of the assumed position (which is the position for which the altitude interpolated for declination is known) and the position circle passing through the zenith of the position for which the altitude is required. This is achieved by the use of auxiliary Diagrams A and B, which are incorporated in each volume of the tables. Diagram A, which consists of a series of graduated parallels and meridians, is used in plotting the zenith of the actual position on Diagram C, in correct relationship to its center, which represents the zenith of the assumed position. Diagram B, which consists of a series of position circles for various altitudes plotted on a graduated center line, is used to place the position circle accurately through the zenith of the position for which the altitude is desired. Diagram C is a loose transparency, a copy of which is supplied with each volume of the tables.

Before effecting the interpolation of altitude for Lat. and LHA, the tabular altitude is to be interpolated to the declination of the observed body. The interpolated respondents then relate to the specific point on Diagram A defined by the intersecting parallel and meridian having the same label as the latitude and local hour angle, respectively, entering into the above solution, i.e., the parallel of the latitude entry, the central meridian $P_0$°.

The point for which the further interpolation is required is the intersection of another parallel and meridian, defined by coordinates differing from those of the former point by the pertinent increments of latitude and local hour angle. Instructions for this interpolation appear in Fig. 4, a composite of Diagrams A, B, and C, which illustrates the following example.

***Example i.*** *Required.*—Determine the altitude and azimuth of *Kochab* at GMT 02$^h$17$^m$34$^s$ in DR latitude 80°28′ N, longitude 162°24′ W.

From *The Nautical Almanac:*

| | | |
|---|---|---|
| GHA and Dec. of *Kochab* | 316°50.4′ | 74°16.7′ N |
| Longitude | 162°24.0′ W | |
| LHA | 154°26.4′ | |

From page 55

| | | Hc | d | Z |
|---|---|---|---|---|
| LHA 154°, Lat. 80°, Dec. 74° (Same) | | 64°38.4′ | (+) 59.0′ | 16.3°† |
| Int. Tab. | Dec. Inc. 16.7′, d 50.0′, 1st part | (+) 13.9′ | | |
| | Dec. Inc. 16.7′, d 9.0′, 2nd part | (+) 2.5′ | | |
| Altitude interpolated for Dec. | | 64°54.8′ | | |
| Corr. from A, B, C, Diagrams (see opposite page) | | (+) 25.6′ | | |
| Altitude interpolated for LHA, Lat. & Dec. | | 65°20.4′ | | Zn 343.7° |
| Altitude computed directly for exact coordinates | | 65°20.5′ | | |

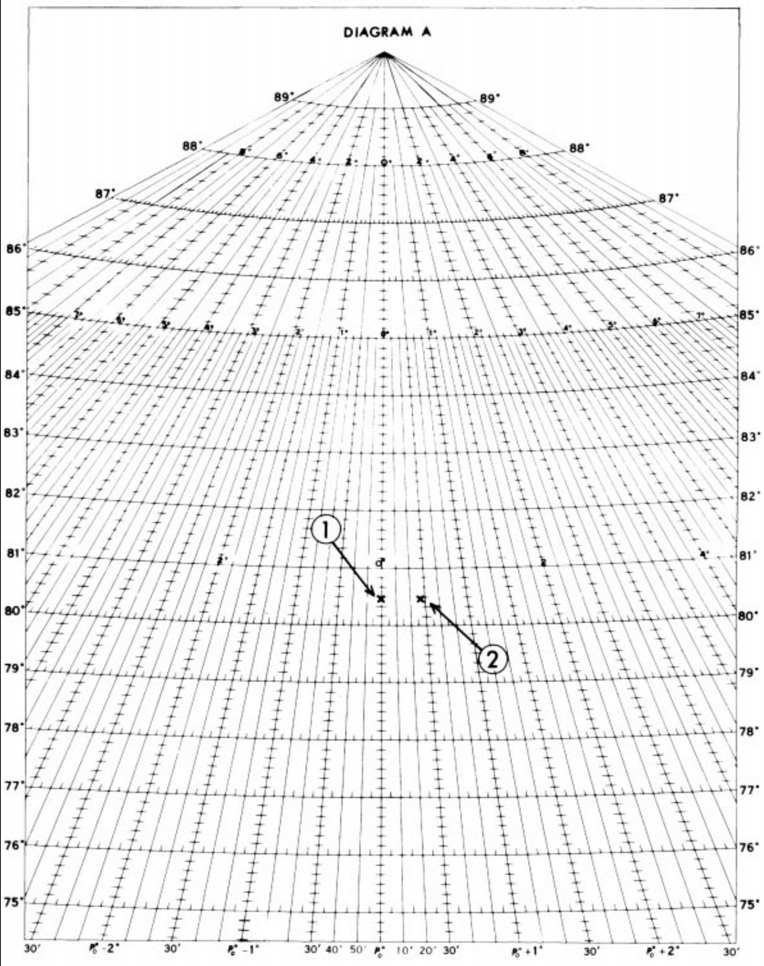
† Interpolated value.

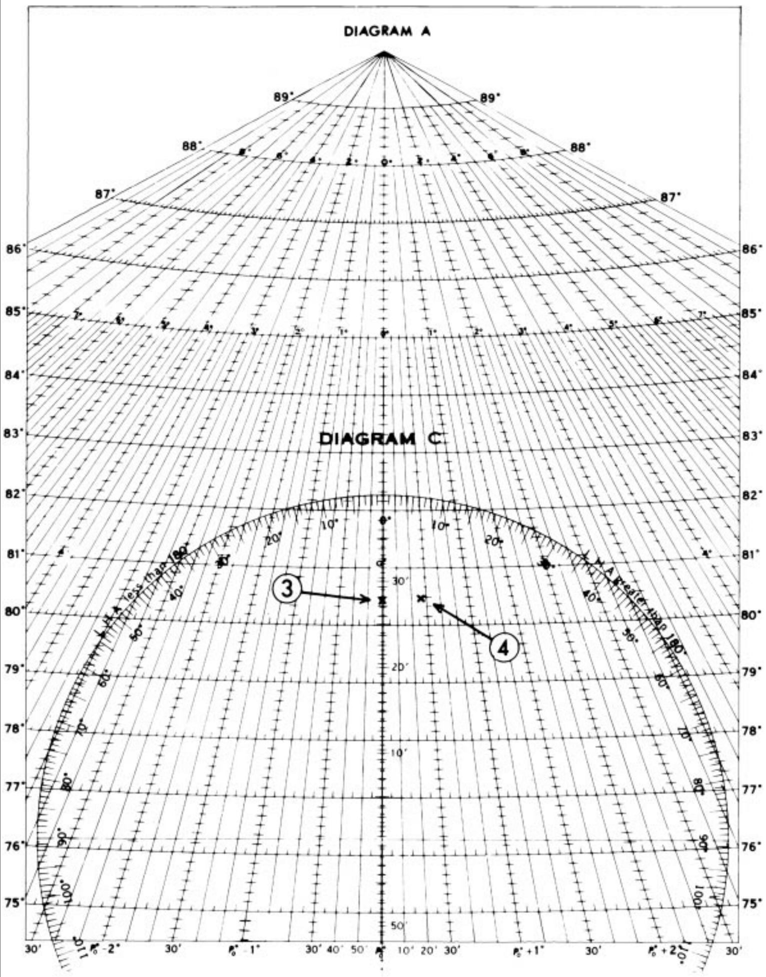
Note that the central meridian of A is taken as 154° (i.e., $P_0$° = 154°); the latitude (1) marked on the central meridian is 80°28′, and the increment of LHA is 26.4′; the intersection of the parallel of 80°28′ and increment of LHA is marked (2). C is superimposed on A so that the central meridians coincide, and the 28′ division on C marked (3) coincides with (1) on A; the position of (2) on A is transferred to C as (4). On the periphery of C the azimuth angle, 16.3°, marked (5), chosen to left of center line because LHA is less than 180°; C is then superimposed on B with the center of C and the indicated azimuth angle immediately over the center line of B; maintaining this orientation, C is moved vertically up or down until (4) falls upon the 65°/53° position line; the distance (7) is that from the center of C to the intersection of the position line with the center line of B. This distance, 25.6′, is the correction to be applied to the altitude to correct for the increments of latitude and LHA—added since the center of C is below the chosen position line.

Instructions for use of Diagrams A, B and C (for brevity these Diagrams are referred to as A, B and C, and ordered numbers of the instruction by their numbers). Altitude and azimuth have already been interpolated for declination

1. Mark the actual latitude on the center line of A.
2. Locate the actual L H A (right or left of the center line) and mark its intersections with the parallel defined by (1).
3. Lay C over A with the center lines in coincidence, and with the odd minutes of the actual latitude indicated on the center line of C immediately over the point (1) marked on A.
4. Trace on C the point of intersection (2) from A.
5. Remove C from A and mark on C the interpolated azimuth angle, choosing the angle to the left of 0° if the L H A is less than 180°, and to the right if greater than 180°.
6. Transfer C to B and with both the center of C and the marked azimuth angle* (5) placed in the toward direction over the center line of B, adjust C vertically to bring (4) on C immediately ove the position circle of B, the range of which includes the interpolated altitude.
7. Measure the distance (intercept) in minutes, along the center line of B, from the position circle used in 6 to the center of C. This is the altitude correction.
8. Apply the distance (7), with the sign indicated on B, to the altitude interpolated for declination and so obtain the altitude Hc interpolated now for all three arguments.

*Should (5) be beyond the range of the center line of Diagram **B** use the supplementary azimuth diametrically opposite.



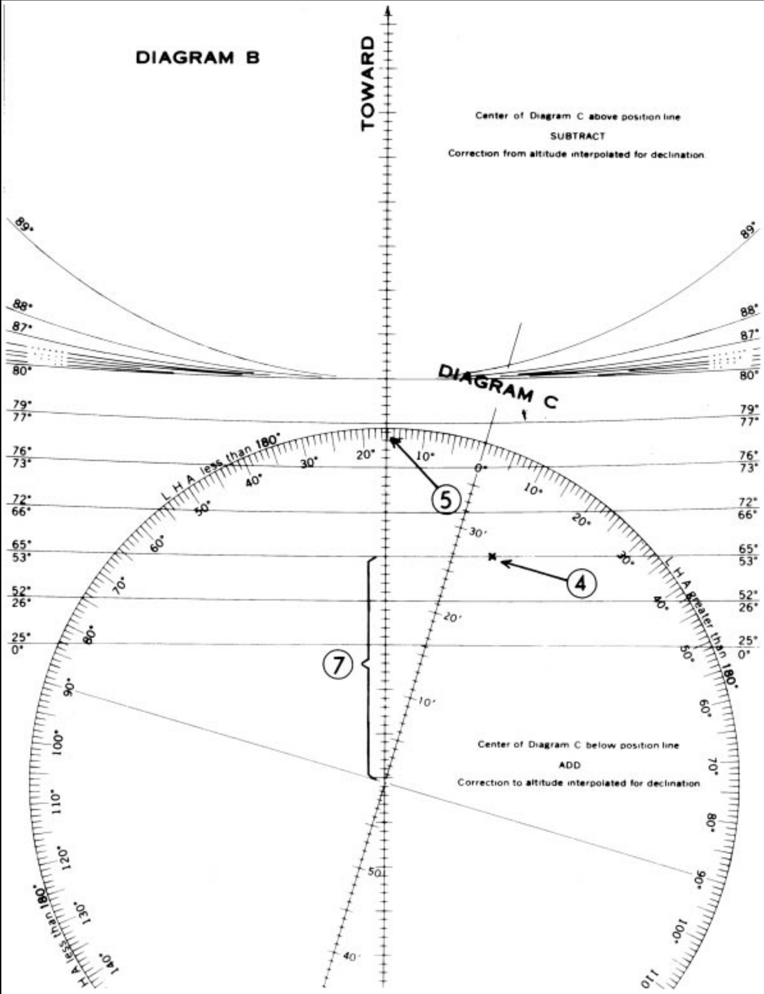




**FIGURE 4**

| | *Example ii* | *Example iii* | *Example iv* |
|---|---|---|---|
| LHA | 330° 35.3′ | 43° 25.0′ | 248° 27.1′ |
| Lat. | 77° 36.3′ S | 85° 27.6′ N | 76° 31.2′ N |
| Dec. | 12° 50.5′ S | 75° 38.7′ N | 85° 57.0′ N |

| | Hc | d | Z | Hc | d | Z | Hc | d | Z |
|---|---|---|---|---|---|---|---|---|---|
| From page 60, 270, 139 | 22°24.6′ | (+)59.7′ | 149.0°† | 78°10.0′ | (+)57.2′ | 119.6°† | 74°26.0′ | (+)36.6′ | 14.6°† |
| Int. Tab. 1st part | (+) 42.1 | | | (+)32.3 | | | (+)28.5 | | |
| Int. Tab. 2nd part | (+) 8.2 | | | (+) 4.6 | | | (+) 6.3 | | |
| Alt. Interpolated for Dec. | 23 14.9 | | | 78 46.9 | | | 75 00.8 | | |
| A, B, C, Dia. Corr. | (+) 17.7 | | | (–) 15.7 | | | (–) 26.3 | | |
| Alt. Interpolated for LHA, Lat., & Dec. | 23 32.6 | | | 78 31.2 | | | 74 34.5 | | |
| Computed values | 23 32.4 | | Zn 031.0 | 78 31.2 | | Zn 240.4 | 74 34.5 | | Zn 014.6 |

† Interpolated value.

In example iv, although the double-second difference of the altitude is 4.9′, the correction is in this case zero.

***Alternative method for interpolating altitude for latitude and local hour angle.***

The table of offsets, page xxii, can also be used to determine graphically the required correction to apply to the altitude at an assumed position to give the altitude at the DR position.

In the diagram (Fig. 5):

D is the DR position.
A is the assumed position.
Zn is the azimuth of the body S.
ED is a segment of the position circle through D.
CD is part of the position line through D.

If *H* is the altitude (interpolated for declination) of S at A, then the altitude at D, which is equal to the altitude at E, is *H*–CA–EC, where CA is the distance of the position line from A and EC is the offset corresponding to the altitude of the body and the distance CD.

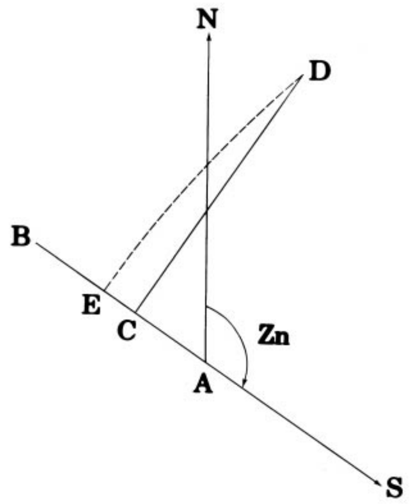



FIGURE 5

*Instructions for use:*

1. Draw the azimuth line from A in the azimuth, Zn.
2. Drop a perpendicular from D to AB at C.

3. Measure CA and CD in nautical miles.
4. From the table of offsets find the offset, EC, corresponding to the altitude, $H$, and the distance CD.
5. Form the altitude at D as $H$ ±CA–EC; note that CA is to be added if C is toward S from A and subtracted if C is away; EC is always to be subtracted.

The above method will give highly satisfactory results except when plotting on a Mercator chart in high latitudes.

**7. Position Circles and Position Lines.** The navigator, working on a Mercator chart or plotting sheet, draws his position line at the extremity of an intercept and at right angles to it. Both the intercept and the position line are drawn as straight lines, rhumb lines on the Mercator projection. The intercept is in reality part of a great circle and the position line is part of a small circle with radius equal to the zenith distance of the observed body. When using these basic sight reduction tables near the equator and interpolating for declination only, the assumed or chosen position can be $(30^2+30^2)^{1/2} = 42'$ from the DR position, so that the intercept can have a maximum value of 42′ (and occasionally may be greater if the DR position is in error) when the position line segment is very short; the position line segment can have a maximum value of 42′, when the intercept is very short; in latitude 75° these maxima reduce to 31′ due to the convergence of the meridians. The error in drawing the intercept (azimuth line) as a rhumb line is negligible except in high latitudes, but the error of drawing the position circle as a rhumb line can reach a maximum of 1.0′ at the equator when the altitude is 75°; in latitude 60° the maximum error for altitude 75° is 0.8′; the error diminishes directly as the tangent of the altitude and does not exceed 0.5′ when the altitude is less than 60°.

The table on page xxii gives the correction to a position line, assumed to be a great circle, to obtain the small circle corresponding to the altitude. The corrections eliminate the major portion of the error of a position line as normally drawn. The table on page xxii gives, for all altitudes and at intervals of 5′ along the position line from the intercept, the offsets required to establish the position circle in lieu of the position line. The offsets should be plotted, as in Fig. 3, from the position line in the direction of the observed body, i.e., perpendicular to the position line—usually two offsets only need be plotted and joined by a straight line to give a portion of the position circle. An extension of this table for polar plotting appears in section C.10, Volume 6 only.

An error of 0.1° in azimuth will introduce a maximum error of 0.07′ in the position line. Hence care should be taken in interpolating the azimuth and plotting the resulting line.

An error of one second in the timing of an observation can produce a maximum error of 0.25′ in the derived position line at the equator, the error depending on the azimuth of the observed body and the latitude of the observer; the error in timing diminishes as the latitude increases, so that in latitude 60° the maximum error in the position line due to an error of one second in timing is 0.12′. Observations made in azimuths near 90° or 270° require the greatest accuracy in timing.

In order to obtain the maximum precision in plotting position lines from assumed (or chosen) positions, consideration should be given to the following points:

(i) Observations should be timed to one quarter of a second if possible.

(ii) A correction for the effect of second differences should be applied when the observed altitude is high (see page xiii).

(iii) A chart or plotting sheet utilizing the stereographic, Lambert conformal, or transverse Mercator projection should be used in preference to the standard Mercator projection, when the latitude is greater than about 60°. Instructions are given on page xxiii for constructing a plotting sheet using Diagram A.

(iv) Position lines should be corrected by means of the offsets tabulated in the table on page xxiv when the observed altitude is very high; these offsets can be applied when plotting on any standard navigational chart.

## TABLE OF OFFSETS (ALTITUDES 0° TO 89°)

| | DISTANCE ALONG LINE OF POSITION FROM INTERCEPT | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 00′ | 05′ | 10′ | 15′ | 20′ | 25′ | 30′ | 35′ | 40′ | 45′ | |
| ***ALT.*** | | | | | **OFFSETS** | | | | | | ***ALT.*** |
| 0° | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0° |
| 30 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 30 |
| 40 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 0.3 | 40 |
| 50 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.2 | 0.3 | 0.3 | 50 |
| 55 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 | 0.4 | 55 |
| 60 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.2 | 0.3 | 0.4 | 0.5 | 60 |
| 62 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.2 | 0.3 | 0.4 | 0.5 | 62 |
| 64 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 64 |
| 66 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.7 | 66 |
| 68 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.3 | 0.4 | 0.6 | 0.7 | 68 |
| 70 | 0.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.2 | 0.4 | 0.5 | 0.6 | 0.8 | 70 |
| 71 | 0.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.7 | 0.9 | 71 |
| 72 | 0.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.7 | 0.9 | 72 |
| 73 | 0.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.6 | 0.8 | 1.0 | 73 |
| 74 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.3 | 0.5 | 0.6 | 0.8 | 1.0 | 74 |
| 75 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.3 | 0.5 | 0.7 | 0.9 | 1.1 | 75 |
| 76 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.4 | 0.5 | 0.7 | 0.9 | 1.2 | 76 |
| 77 | 0.0 | 0.0 | 0.1 | 0.1 | 0.3 | 0.4 | 0.6 | 0.8 | 1.0 | 1.3 | 77 |
| 78 | 0.0 | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.6 | 0.8 | 1.1 | 1.4 | 78 |
| 79 | 0.0 | 0.0 | 0.1 | 0.2 | 0.3 | 0.5 | 0.7 | 0.9 | 1.2 | 1.5 | 79 |
| 80.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.3 | 0.5 | 0.7 | 1.0 | 1.3 | 1.7 | 80.0 |
| 80.5 | 0.0 | 0.0 | 0.1 | 0.2 | 0.3 | 0.5 | 0.8 | 1.1 | 1.4 | 1.8 | 80.5 |
| 81.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1.1 | 1.5 | 1.9 | 81.0 |
| 81.5 | 0.0 | 0.0 | 0.1 | 0.2 | 0.4 | 0.6 | 0.9 | 1.2 | 1.6 | 2.0 | 81.5 |
| 82.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.4 | 0.6 | 0.9 | 1.3 | 1.7 | 2.1 | 82.0 |
| 82.5 | 0.0 | 0.0 | 0.1 | 0.2 | 0.4 | 0.7 | 1.0 | 1.4 | 1.8 | 2.2 | 82.5 |
| 83.0 | 0.0 | 0.0 | 0.1 | 0.3 | 0.5 | 0.7 | 1.1 | 1.5 | 1.9 | 2.4 | 83.0 |
| 83.5 | 0.0 | 0.0 | 0.1 | 0.3 | 0.5 | 0.8 | 1.2 | 1.6 | 2.0 | 2.6 | 83.5 |
| 84.0 | 0.0 | 0.0 | 0.1 | 0.3 | 0.5 | 0.9 | 1.2 | 1.7 | 2.2 | 2.8 | 84.0 |
| 84.5 | 0.0 | 0.0 | 0.2 | 0.3 | 0.6 | 1.0 | 1.4 | 1.9 | 2.4 | 3.1 | 84.5 |
| 85.0 | 0.0 | 0.0 | 0.2 | 0.4 | 0.7 | 1.0 | 1.5 | 2.1 | 2.7 | 3.4 | 85.0 |
| 85.5 | 0.0 | 0.0 | 0.2 | 0.4 | 0.7 | 1.2 | 1.7 | 2.3 | 3.0 | 3.8 | 85.5 |
| 86.0 | 0.0 | 0.1 | 0.2 | 0.5 | 0.8 | 1.3 | 1.9 | 2.6 | 3.4 | 4.3 | 86.0 |
| 86.5 | 0.0 | 0.1 | 0.2 | 0.5 | 1.0 | 1.5 | 2.2 | 2.9 | 3.8 | 4.9 | 86.5 |
| 87.0 | 0.0 | 0.1 | 0.3 | 0.6 | 1.1 | 1.7 | 2.5 | 3.4 | 4.5 | 5.7 | 87.0 |
| 87.5 | 0.0 | 0.1 | 0.3 | 0.8 | 1.3 | 2.1 | 3.0 | 4.1 | 5.4 | 6.9 | 87.5 |
| 88.0 | 0.0 | 0.1 | 0.4 | 0.9 | 1.7 | 2.7 | 3.8 | 5.2 | 6.9 | 8.8 | 88.0 |
| 88.5 | 0.0 | 0.2 | 0.6 | 1.3 | 2.3 | 3.5 | 5.1 | 7.1 | 9.4 | 12.1 | 88.5 |
| 89.0 | 0.0 | 0.3 | 0.8 | 1.9 | 3.4 | 5.5 | 8.0 | 11.3 | 15.3 | 20.3 | 89.0 |

The distance is measured along the position line from the intercept. The offsets are to be drawn at right angles to the position line in the direction of the observed body. Join the extremities of the offsets to obtain the position circle.

**8. Construction of a Plotting Sheet.** Although essential navigational charts and plotting sheets normally will be available, it may become necessary for the navigator to construct an accurate graticule of a particular area for immediate use. The following instructions provide a rapid method of construction utilizing Diagram A. Each parallel of this diagram is projected as if the plane of projection were tangent at that parallel; hence these parallels may be used to establish an accurate small area plotting sheet, resembling the stereographic projection.

*Instructions*—Draw a straight line to form the central meridian on a suitable sheet of paper and mark intervals of six inches along this line (these will be the points through which the parallels for successive integral degrees of latitude must be drawn); place a piece of tracing paper over Diagram A in the area required, and mark the intersections of the meridians and parallels; transfer the tracing to the construction sheet and superimpose the tracing paper over the central meridian, with each parallel occupying successively the points through which it is desired to draw that parallel and "mark through" sufficient intersections of each parallel with successive meridians. The indicated points can then be connected to establish the parallels, and the meridians can be drawn in as straight lines.

When using the plotting sheet, all azimuths must be measured from the local meridian; the scale will be 10 nautical miles to one inch.

**9. Variation of Altitude with Time.** Inspection of successive altitude differences in the basic tables reveals immediately the change in altitude due to a change of one degree in the declination argument. Frequently navigators desire to ascertain the change with time of the altitude of a celestial body. The difference between the tabular values of the altitude given for the same arguments of latitude and declination, but differing by one degree in LHA, indicates the change of the altitude over a period of four minutes of time. In general the latitude and declination need only be used to the nearest integral degree in order to obtain a good approximation of the rate of change. To determine this approximate rate of change with time, enter the tables with integral degrees of LHA, latitude, and declination, and extract the altitude; turn to the next opening—next value of LHA—and with the same arguments for latitude and declination, extract the corresponding altitude; the difference between the two altitudes divided by four is the rate of change of altitude of the body in one minute of time.

**10. Polar Plotting.** For navigation within a few degrees of the pole it is convenient to adopt the pole as the assumed position and plot from that geographic position or point. At the pole the declination of the observed body, and the calculated altitude, Hc, coincide, and the GHA at the time of observation coincides with the azimuth of the observed body, measured westward from the Greenwich meridian; the intercept is thus the difference between the observed altitude, Ho, and the declination. Thus the position line may be constructed without extracting data from basic sight reduction tables. This method, however, frequently involves the use of long position lines which must be modified if accurate results are required.

The table on page xxiv gives offsets in minutes of are (nautical miles) for observed altitudes to 70° at distances to 300 minutes of arc along the position line from the intercept; the offset for any altitude and any point along the position line is the distance separating the position line from the position circle; the offsets are to be plotted perpendicular to the position line and toward the observed body. Normally only two offsets, in the vicinity of the DR position, need be plotted and their extremities joined by a straight line, to give a very good approximation to that part of the position circle.

The following example illustrates the calculation and plotting procedure for a typical sight when:

(i) the pole is used as the assumed position;

(ii) an assumed position is chosen in the usual way.

## TABLE OF OFFSETS (ALTITUDES 0° TO 70°)

***DISTANCE ALONG LINE OF POSITION FROM INTERCEPT***

| ALT. | 20′ | 40′ | 60′ | 80′ | 100′ | 120′ | 140′ | 160′ | 180′ | 200′ | 220′ | 240′ | 260′ | 280′ | 300′ | ALT. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | OFFSETS | | | | | | | | | | | | | | | |
| 0° | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0.0′ | 0° |
| 5 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.2 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.9 | 1.0 | 1.1 | 5 |
| 10 | 0.0 | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.7 | 0.8 | 1.0 | 1.2 | 1.5 | 1.7 | 2.0 | 2.3 | 10 |
| 15 | 0.0 | 0.1 | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.3 | 1.6 | 1.9 | 2.2 | 2.6 | 3.1 | 3.5 | 15 |
| 20 | 0.0 | 0.1 | 0.2 | 0.3 | 0.5 | 0.8 | 1.0 | 1.4 | 1.7 | 2.1 | 2.6 | 3.1 | 3.6 | 4.2 | 4.8 | 20 |
| 25 | 0.0 | 0.1 | 0.2 | 0.4 | 0.7 | 1.0 | 1.3 | 1.7 | 2.2 | 2.7 | 3.3 | 3.9 | 4.6 | 5.3 | 6.1 | 25 |
| 30 | 0.0 | 0.1 | 0.3 | 0.5 | 0.8 | 1.2 | 1.6 | 2.1 | 2.7 | 3.4 | 4.1 | 4.8 | 5.7 | 6.6 | 7.6 | 30 |
| 35 | 0.0 | 0.2 | 0.4 | 0.7 | 1.0 | 1.5 | 2.0 | 2.6 | 3.3 | 4.1 | 4.9 | 5.9 | 6.9 | 8.0 | 9.2 | 35 |
| 40 | 0.0 | 0.2 | 0.4 | 0.8 | 1.2 | 1.8 | 2.4 | 3.2 | 4.0 | 4.9 | 6.0 | 7.1 | 8.4 | 9.7 | 11.1 | 40 |
| 45 | 0.1 | 0.2 | 0.5 | 0.9 | 1.5 | 2.1 | 2.9 | 3.7 | 4.7 | 5.8 | 7.0 | 8.4 | 9.8 | 11.4 | 13.1 | 45 |
| 50 | 0.1 | 0.3 | 0.6 | 1.1 | 1.7 | 2.5 | 3.4 | 4.4 | 5.6 | 6.9 | 8.4 | 10.0 | 11.7 | 13.6 | 15.6 | 50 |
| 52 | 0.1 | 0.3 | 0.7 | 1.2 | 1.9 | 2.7 | 3.7 | 4.8 | 6.0 | 7.5 | 9.0 | 10.7 | 12.6 | 14.6 | 16.8 | 52 |
| 54 | 0.1 | 0.3 | 0.7 | 1.3 | 2.0 | 2.9 | 3.9 | 5.1 | 6.5 | 8.0 | 9.6 | 11.6 | 13.6 | 15.7 | 18.1 | 54 |
| 56 | 0.1 | 0.3 | 0.8 | 1.4 | 2.2 | 3.1 | 4.2 | 5.5 | 7.0 | 8.6 | 10.5 | 12.5 | 14.6 | 17.0 | 19.5 | 56 |
| 58 | 0.1 | 0.4 | 0.8 | 1.5 | 2.3 | 3.4 | 4.6 | 6.0 | 7.6 | 9.3 | 11.3 | 13.5 | 15.8 | 18.3 | 21.1 | 58 |
| 60 | 0.1 | 0.4 | 0.9 | 1.6 | 2.5 | 3.6 | 4.9 | 6.5 | 8.2 | 10.1 | 12.2 | 14.6 | 17.1 | 19.8 | 22.8 | 60 |
| 61 | 0.1 | 0.4 | 0.9 | 1.7 | 2.6 | 3.8 | 5.1 | 6.7 | 8.5 | 10.5 | 12.7 | 15.2 | 17.8 | 20.7 | 23.8 | 61 |
| 62 | 0.1 | 0.4 | 1.0 | 1.8 | 2.7 | 3.9 | 5.4 | 7.0 | 8.9 | 11.0 | 13.3 | 15.8 | 18.6 | 21.6 | 24.8 | 62 |
| 63 | 0.1 | 0.5 | 1.0 | 1.8 | 2.9 | 4.1 | 5.6 | 7.3 | 9.3 | 11.5 | 13.9 | 16.5 | 19.4 | 22.5 | 25.9 | 63 |
| 64 | 0.1 | 0.5 | 1.1 | 1.9 | 3.0 | 4.3 | 5.9 | 7.7 | 9.7 | 12.0 | 14.5 | 17.3 | 20.3 | 23.5 | 27.1 | 64 |
| 65 | 0.1 | 0.5 | 1.1 | 2.0 | 3.1 | 4.5 | 6.1 | 8.0 | 10.1 | 12.5 | 15.2 | 18.1 | 21.2 | 24.6 | 28.3 | 65 |
| 66 | 0.1 | 0.5 | 1.2 | 2.1 | 3.3 | 4.7 | 6.4 | 8.4 | 10.6 | 13.1 | 15.9 | 18.9 | 22.2 | 25.8 | 29.7 | 66 |
| 67 | 0.1 | 0.5 | 1.2 | 2.2 | 3.4 | 4.9 | 6.7 | 8.8 | 11.1 | 13.8 | 16.7 | 19.9 | 23.4 | 27.1 | 31.2 | 67 |
| 68 | 0.1 | 0.6 | 1.3 | 2.3 | 3.6 | 5.2 | 7.1 | 9.2 | 11.7 | 14.5 | 17.5 | 20.9 | 24.6 | 28.5 | 32.8 | 68 |
| 69 | 0.2 | 0.6 | 1.4 | 2.4 | 3.8 | 5.5 | 7.4 | 9.7 | 12.3 | 15.2 | 18.5 | 22.0 | 25.9 | 30.0 | 34.5 | 69 |
| 70 | 0.2 | 0.6 | 1.4 | 2.6 | 4.0 | 5.8 | 7.9 | 10.3 | 13.0 | 16.1 | 19.5 | 23.2 | 27.3 | 31.7 | 36.5 | 70 |

The distance is measured along the position line from the intercept. The offsets are to be drawn at right angles to the position line in the direction of the observed body. Join the extremities of the offsets to obtain the position circle.

***Example***—On a certain date in DR position 88°20′ S, 66°40′ E, the following observations of two stars were made. At GMT $12^h42^m34^s$ the observed altitude of *Canopus* was 51°15.6′; at $12^h47^m49^s$ the observed altitude of *Achernar* was 56°06.1′.

| | | *Canopus* | | | *Achernar* | |
|---|---|---|---|---|---|---|
| GHA♈ | $12^h$ | 165°38.4′ | | $12^h$ | 165°38.4′ | |
| Inc. | $42^m34^s$ | 10 40.2 | | $47^m49^s$ | 11 59.2 | |
| SHA, Dec. | | 264 11.4 | 52°40.2′ S | | 335 51.1 | 57°23.4′ S |
| GHA☆ | $12^h42^m34^s$ | 80 30.0 | | $12^h47^m49^s$ | 153 28.7 | |

(i) For the South Pole as the assumed position.

| | | | | |
|---|---|---|---|---|
| GHA | (Long. W) | 80°30.0′ | (Long. W) | 153°28.7′ |
| Dec. | | 52 40.2 S | | 57 23.4 S |
| Ho | | 51 15.6 | | 56 06.1 |
| Intercept | | 1 24.6 A | | 1 17.3 A |

(ii) For the normal method, assumed latitude = 88° S

| | | |
|---|---|---|
| GHA☆ | 80°30.0′ | 153°28.7′ |
| aλ | 66°30.0′ E | 66°31.3′ E |
| LHA | 147 | 220 |

| | Hc | d | Z | | Hc | d | Z |
|---|---|---|---|---|---|---|---|
| From page 251 | 50°18.6′ | (+)60.0′ | 31.7°† | page 265 | 55°26.8′ | (+)60.0′ | 38.1°† |
| Alt. Corr. | (+) 40.2* | | | | (+) 23.4* | | |
| Hc | 50 58.8 | | | | 55 50.2 | | |
| Ho | 51 15.6 | | | | 56 06.1 | | |
| Intercept | 16.8 T | Zn 211.7 | | | 15.9 T | Zn 141.9 | |

*When d = 60.0′ the use of the Interpolation Table is not required, for the change in altitude is equal to the change in declination, hence the altitude correction is equal to the declination increment. †Interpolated value.

The plotting sheet portraying these data was constructed using Diagram A, then reduced to scale.

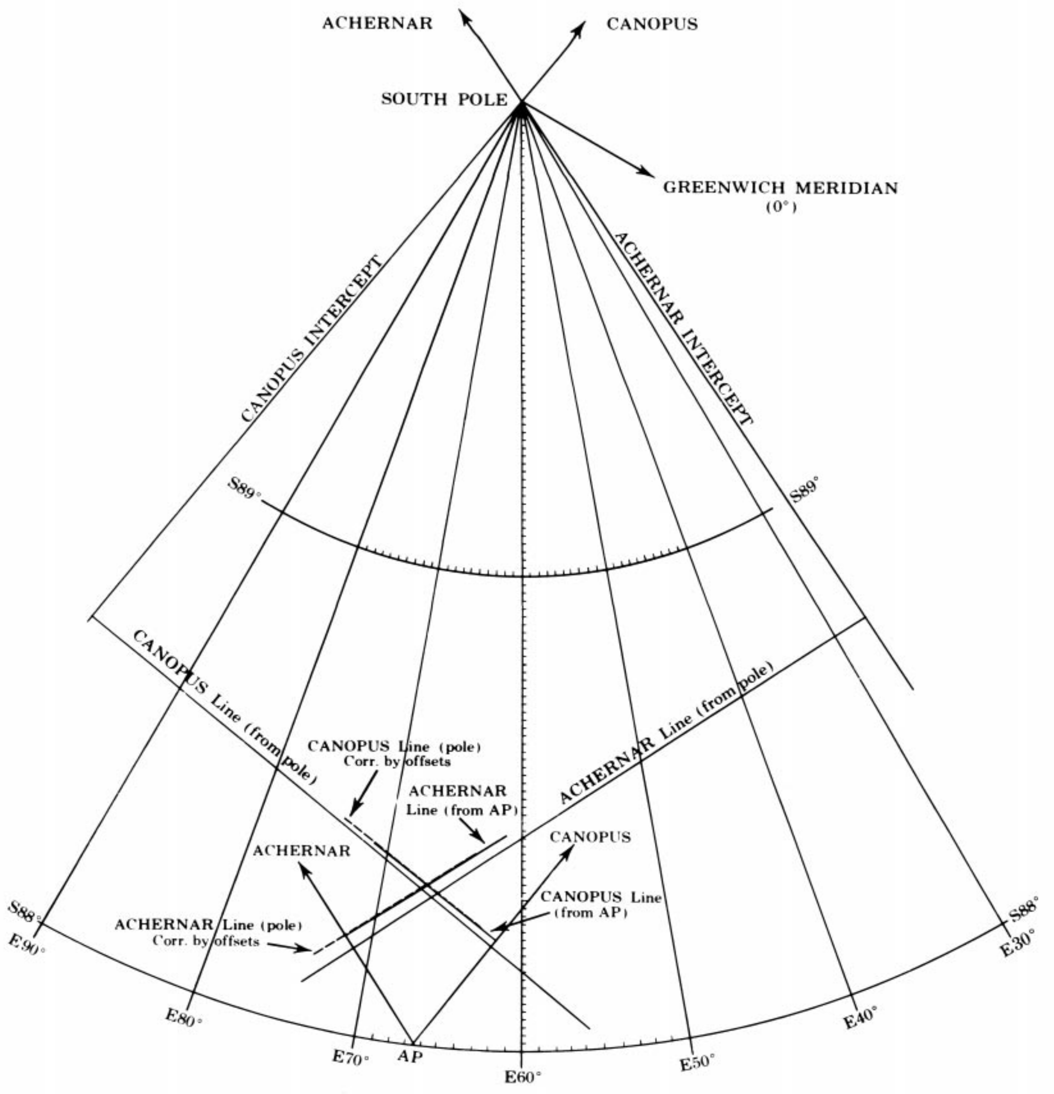



**Figure 6—Plot from pole using offsets gives same fix as normal plot**.

**11. The Polar Grid.** Because of the rapid convergence of meridians in polar areas, polar charts are often overprinted with a series of parallel lines called a GRID; by convention they are drawn parallel to the Greenwich meridian. Grid North is taken to be the direction north along the Greenwich meridian to the north pole, then south along the 180° meridian to the south pole.

A straight line on a polar chart has the same direction with reference to all the grid lines and therefore represents a constant grid direction.

The direction of grid north at any point differs from true north by the longitude of the point and is measured from TRUE NORTH as follows:

| *LONGITUDE* | *N. HEMISPHERE* | *S. HEMISPHERE* |
|---|---|---|
| EAST | CLOCKWISE | COUNTERCLOCKWISE |
| WEST | COUNTERCLOCKWISE | CLOCKWISE |

If T represents the true direction and G the grid direction, these can be converted from one to the other as follows:

| *N. HEMISPHERE* | *S. HEMISPHERE* |
|---|---|
| G=T+Long. W | G=T–Long. W |
| G=T–Long. E | G=T+Long. E |
| T=G–Long. W | T=G+Long. W |
| T=G+Long. E | T=G–Long. E |

The above formulas hold true only when the convergency is 1.0, but can generally be considered accurate enough for navigational purposes in polar latitudes.

## D. OTHER APPLICATIONS

**1. Star Identification**. Since many navigators either know their stars or have preferred means of identification, including the use of star finders, star charts, star globes, etc., no formal star identification tables are included in these volumes. Nevertheless, in cloud-covered skies, an unidentified star may be the only body visible for positioning, and it then becomes necessary to identify it from its observed altitude and azimuth. A simple approach to star identification, from a knowledge of observed altitude and azimuth, is to scan the pages of the appropriate latitudes of the specific volume of these tables and observe the combination of arguments which give the altitude and azimuth of the observation. Thus the declination and LHA☆ is directly determined. From these quantities the star can be identified from *The Nautical Almanac*.

In view of the navigator's knowledge of the date and time of observation and his approximate position (latitude and longitude), another solution is available through an interchange of arguments. The procedure consists of entering the tables with the known latitude, with the observed azimuth as LHA, and the observed altitude as declination, and extracting from the tables, for these arguments, the altitude and azimuth respondents; the extracted altitude becomes the body's declination, the extracted azimuth the body's local hour angle. The tables should always be entered with integral declination of same name, nearest the observed altitude, although the respondents may come from either same or contrary name areas; in the latter case the required declination is of contrary name. The star's SHA is found from SHA☆=LHA☆–LHA♈. The tabular arrangement, on page xxvii, offers guidance for the solution which is required only to the nearest integral degree.

## D. OTHER APPLICATIONS

| Entering arguments | | | | | Star coordinates | |
|---|---|---|---|---|---|---|
| LAT. | LHA | DEC. | Range of Obs. Az. | Location of Hc and Z | DEC. | LHA |
| | | | NORTH LATITUDE | | | |
| | | | 0° to 90° | Left page | Hc (North) | 360°– Z |
| Lat. | Obs. Az. | Obs. Alt. | 90° to 180° | Right page, below C-S Line | Hc (North) | 360°– Z |
| | | | 90° to 180° | Right page, above C-S Line | Hc (South) | 180° +Z |
| | | | 180° to 270° | Right page, below C-S Line | Hc (North) | Z |
| Lat. | Obs. Az. | Obs. Alt. | 180° to 270° | Right page, above C-S Line | Hc (South) | 180° –Z |
| | | | 270° to 360° | Left page | Hc (North) | Z |
| | | | SOUTH LATITUDE | | | |
| | | | 0° to 90° | Right page, below C-S Line | Hc (South) | 360°– Z |
| Lat. | 180°–Obs. Az. | Obs. Alt. | 0° to 90° | Right page, above C-S Line | Hc (North) | 180° +Z |
| | | | 90° to 180° | Left page | Hc (South) | 360°– Z |
| | | | 180° to 270° | Left page | Hc (South) | Z |
| Lat. | Obs. Az.–180° | Obs. Alt. | 270° to 360° | Right page, below C-S Line | Hc (South) | Z |
| | | | 270° to 360° | Right page, above C-S Line | Hc (North) | 180° –Z |

## EXAMPLES FOR STAR IDENTIFICATION (Selection for illustration only)

| Ex. | Lat. | Long. | Obs. Alt. | Obs. Az. | LHAϒ* |
|---|---|---|---|---|---|
| 1 | 77° 15′ N | 33° 55′ W | 37° 41′ | 80° | 189° |
| 2 | 75 54 N | 168 10 E | 31 54 | 98 | 288 |
| 3 | 75 06 N | 143 40 W | 7 02 | 166 | 64 |
| 4 | 80 38 N | 7 27 W | 27 25 | 206 | 239 |
| 5 | 76 22 N | 10 14 E | 5 10 | 202 | 100 |
| 6 | 78 43 N | 16 42 E | 33 19 | 352 | 121 |
| 7 | 80 55 S | 165 19 W | 38 07 | 23 | 324 |
| 8 | 75 28 S | 77 33 E | 9 58 | 20 | 25 |
| 9 | 79 12 S | 14 02 W | 50 00 | 137 | 258 |
| 10 | 87 43 S | 49 17 E | 56 35 | 254 | 50 |
| 11 | 76 04 S | 14 22 W | 17 25 | 296 | 262 |
| 12 | 75 24 S | 77 14 E | 6 49 | 338 | 110 |

*LHAϒ from *The Nautical Almanac* for date and GMT of observation.

## SOLUTIONS

| | Entering Arguments | | | | Star Coordinates and Identity | | | |
|---|---|---|---|---|---|---|---|---|
| Ex. | Lat. | LHA | Dec. | Page | Dec. | LHA | SHA | Name |
| 1 | 77° | 80° | 38° | Left | 39° N | 360°– 90° =270° | 81° | *Vega* |
| 2 | 76 | 98 | 32 | Right, below C-S Line | 29 N | 360 – 74 =286 | 358 | *Alpheratz* |
| 3 | 75 | 166 | 7 | Right, above C-S Line | 8 S | 180 + 166 =346 | 282 | *Rigel* |
| 4 | 81 | 206 | 27 | Right, below C-S Line | 19 N | 24 | 145 | *Arcturus* |
| 5 | 76 | 202 | 5 | Right, above C-S Line | 8 S | 180 – 158 =22 | 282 | *Rigel* |
| 6 | 79 | 352 | 33 | Left | 44 N | 171 | 50 | *Deneb* |
| 7 | 81 | 180–23=157 | 38 | Right, below C-S Line | 30 S | 360 – 21 =339 | 15 | *Fomalhaut* |
| 8 | 75 | 180–20=160 | 10 | Right, above C-S Line | 4 N | 180 + 160 =340 | 315 | *Menkar* |
| 9 | 79 | 180–137=43 | 50 | Left | 57 S | 360 – 126 =234 | 336 | *Achernar* |
| 10 | 88 | 254–180=74 | 57 | Left | 57 S | 103 | 53 | *Peacock* |
| 11 | 76 | 296–180= 116 | 17 | Right, below C-S Line | 11 S | 61 | 159 | *Spica* |
| 12 | 75 | 338–180= 158 | 7 | Right, above C-S Line | 7 N | 180 – 158 = 22 | 272 | *Betelgeuse* |

SHA☆=LHA☆–LHAϒ

**2. Great-Circle Sailing.** The great-circle distance between any two points on the spherical surface of the Earth and the initial great-circle course may be found by relating the problem to the solution of the navigational triangle, which solution is always available from these tables. For by entering the tables with latitude of departure as latitude, latitude of destination as declination, and difference of longitude as LHA, the altitude and azimuth angle may be extracted and converted to distance and course. The instructions below provide details of the modifications necessary to the tabular entries and the extracted data.

The tabular azimuth angle (or its supplement) becomes the initial great-circle course, named N or S for the latitude of departure, and E or W depending upon the destination being E or W of point of departure.

If all entering arguments are integral degrees, the altitude and azimuth angle are obtained directly from the tables, without interpolation. If the latitude of destination is nonintegral, interpolation for the additional minutes of latitude is done as in correcting altitude for any declination increment; if either the latitude of departure or difference in longitude, or both, are nonintegral, interpolate graphically using Diagrams A, B, and C and apply this correction additionally, as in correcting the altitude for any DR or other selected position.

Since, in the great-circle-distance solution, the latitude of destination becomes the declination entry, and all declinations appear on every page, the great-circle distance can always be found from the volume which covers the latitude belt containing the latitude of departure. Great-circle solutions belong in one of the four following categories.

It is always known if latitudes of departure and destination are of same or contrary name. In those instances when it is not known whether the great-circle distance is less than or greater than 90° (5400 nautical miles), test first Case I for latitudes of same name and Case II for latitudes of contrary name. Failing solution by these rules, consider Cases III and IV, respectively.

*Case I*—Latitudes of departure and destination of same name and initial great-circle distance less than 90°.

Enter the tables with latitude of departure as latitude entry, latitude of destination as declination entry of same name, and difference of longitude as local hour angle entry; extract the tabular altitude, which subtracted from 90° is the desired great-circle (zenith) distance. The azimuth angle is the initial great-circle course angle. If no such entry can be found, the distance is greater than 90°.

*Case II*—Latitudes of departure and destination of contrary name and great-circle distance less than 90°.

Enter tables with latitude of departure as latitude entry and latitude of destination as declination entry of contrary name, and with the difference of longitude as local hour angle entry; extract the tabular altitude, which subtracted from 90° is the required distance. The azimuth angle is the initial great-circle course angle. If no such entry can be found the distance is greater than 90°.

*Case III*—Latitudes of departure and destination of same name and great-circle distance greater than 90°.

Enter the tables with latitude of departure as latitude entry, latitude of destination as declination entry of opposite name to latitude of destination, and with 180° minus longitude difference as local hour angle; extract the altitude which added to 90° gives the required distance. The initial great-circle course angle is 180° minus the azimuth angle.

*Case IV*—Latitudes of departure and destination of contrary name and great-circle distance greater than 90°.

Enter tables with latitude of departure as latitude entry, and latitude of destination as declination entry of opposite name to latitude of destination, and with 180° minus longitude difference as local hour angle; extract the altitude which added to 90° is the required distance. The initial great-circle course angle is 180° minus the azimuth angle.

The following two great-circle distance and course examples, together with their solutions, illustrate Cases I and IV.

***Example i***. *Required*.—Distance and initial great-circle course from Barentsburg 78°04′ N, 14°14′ E to Straumner 66°25′ N, 23°08′ W.

Enter the tables with 78° as the latitude argument, the difference in longitude between 14°14′ E and 23°08′ W is 37°22′— use 37° as the LHA argument, and 66° as the declination argument (same name).

| | | Hc | d | Z |
|---|---|---|---|---|
| From page 76 | | Hc | d | Z |
| LHA 37°, Lat. 78°, Dec. 66° | | 73°58.2′ | (+) 53.1′ | 116.8°† |
| Dec. Inc. 25′, d+53.1′ | 1st part | (+) 20.8 | | |
| | 2nd part | (+) 1.3 | | |
| Interpolated for Dec. Inc. | | 74 20.3 | | Zn 243.2° |
| †Interpolated value. | | | | |

Using the A, B, C Diagrams to interpolate for latitude and local hour angle (difference in longitude) increments, the correction to Hc is –6.0′, so that the corrected Hc = 74°14.3′, therefore the distance (90° – Hc) = 15°45.7′ = 945.7 nautical miles and the initial great-circle course is 243.2°.

***Example ii.*** *Required*.—Distance and initial great-circle course from Cape Bird 77°08′ S, 166°30′ E to San Francisco 37°49' N, 122°25′ W.

Enter the tables with latitude 77° as latitude argument, the difference in longitude between 166°30′ E and 122°25′ W is 71°05′—use 71° as LHA argument, and 37° as declination argument (of contrary name). It is found that these arguments are not available, so find the supplement 180° – LHA = 108°55′ (use 109°) for LHA argument, with declination changed to same name.

| | | | | |
|---|---|---|---|---|
| From page 145 | | Hc | d | Z |
| LHA 109°, Lat. 77°, Dec. 37° | | 31°51.8′ | (+) 58.1′ | 62.6°† |
| Dec. Inc. 49′, d+58.1′ | 1st part | (+) 40.8 | | 180°– Z = S 117.4° E |
| | 2nd part | (+) 6.7 | | |
| Interpolated for Dec. Inc. | | 32 39.3 | | Zn 062.6° |
| †Interpolated value. | | | | |

Using the A, B, C Diagrams to interpolate for latitude and local hour angle (difference in longitude) increments (note that in this case interpolation is for LHA 108°55′); the correction to Hc is +4.8′, hence the corrected Hc = 32°44.1′ and the distance = 90°+32°44.1′ = 122°44.1′ = 7364.1 nautical miles and the initial great-circle course is 062.6°.

**3. Composite Sailing.** The data from these tables are applicable to the rapid solution of problems of composite sailing, an extension of great-circle sailing. The composite track is frequently the shortest possible safe track, and usually consists of two great-circle arcs and the small-circle arc to which they are tangent. Expressed otherwise, the vertices of the great circle passing through the point of departure and of that passing through the point of destination lie upon the limiting parallel. The complete solution consists of finding the combined length of two great-circle arcs and the length of the intervening parallel. The latter is a problem in parallel sailing.

To effect solutions of the first great-circle distance, enter the tables with LHA = 90°, and with the latitude of the limiting parallel, and find that declination for which the altitude is equal to the latitude of departure; then 90° minus declination is equal to the distance from the point of departure to the point of tangency of the great circle with the parallel of limiting latitude, and the azimuth angle is the difference in longitude between the point of departure and the point of tangency. At the same opening find the corresponding quantities from the limiting parallel to the destination.

The course from point of departure can be found in two ways:

(i) Enter the tables with 90° as LHA, 90° minus distance to point of tangency as latitude, and with the limiting latitude as declination; the course is then the azimuth angle obtained from the tables, the angle being measured from the elevated pole.

(ii) Enter the tables with the difference in longitude (between point of departure and point of tangency) as LHA, latitude of departure as latitude argument, and latitude of the limiting parallel as declination argument; the course angle is then the azimuth angle as before, and 90° minus the altitude is the distance, which serves to check the previous determination. The distance along the limiting parallel is the difference in longitude between the two points of tangency multiplied by the cosine of the latitude (parallel sailing); this difference in longitude is the difference in longitude between points of departure and destination, less the sum of the differences in longitude between the point of departure and the first point of tangency, and between the second point of tangency and the destination.

***Example.*** It is required to determine the component parts of a composite track between Valparaiso 33°02′ S, 71°37′ W and Coulman Island 73°20′ S, 170°10′ E when the limiting latitude is 75° S; it is also desired to plot the course from Valparaiso to the limiting latitude.

Enter the tables with LHA argument as 90°, with latitude argument as 75°, same name; find the declination which corresponds to an altitude, Hc, of 33.0°; the declination is found by mental interpolation to be 34.3° and the azimuth angle is 80.0°. The distance from Valparaiso to the first point of tangency is therefore 90° – 34.3° = 55.7° and the difference in longitude is 80.0°, therefore the longitude of first point of tangency is 71.6°W + 80.0° = 151.6° W. In the same column the declination and azimuth angle corresponding to an altitude of 73.3°, i.e., latitude of Coulman Island are found as 82.6° and 26.6°, respectively; the distance from the second point of tangency to Coulman Island is therefore 90° – 82.6° = 7.4°, and the longitude of the second point of tangency is 170.2°E + 26.6° = 163.2°W. The difference in longitude between the two points of tangency is then 163.2° – 151.6° = 11.6°; the distance along the 75° S parallel is therefore 11.6° cos 75° = 11.6° × 0.259 = 3.0°. The total sailing distance is therefore 55.7° + 3.0° + 7.4° = 66.1° or 3966 nautical miles, approximately.

The course angle from Valparaiso is found by entering with 90° – 55.7° = 34.3° as latitude argument, 90° as LHA argument, and 75° as declination argument; the azimuth angle interpolated mentally is then 18.0°, measured from the south.

Required are a number of points on the great-circle course between Valparaiso and the first point of tangency. In order to illustrate the use of this volume the points will be found with reference to the tangent point 75° S, 151.6° W; the points for plotting will be determined at 600 mile intervals, i.e., at arc distances of 10°, 20°, … from the latter point.

Enter the tables with latitude 75°, LHA 90°, and with successive declinations of 80°, 70°, … the latitudes and differences in longitude from first point of tangency are found as tabulated altitudes and azimuth angles, respectively. The results are as follows:

| Distance n.mi. (arc) | 600 (10°) | 1200 (20°) | 1800 (30°) | 2400 (40°) | 3000 (50°) |
|---|---|---|---|---|---|
| Latitude | 72.0 S | 65.2 S | 56.8 S | 47.7 S | 38.4 S |
| Diff. in Long. | 34.3 | 54.6 | 65.9 | 72.9 | 77.7 |
| Longitude | 117.3 W | 97.0 W | 85.7 W | 78.7 W | 73.9 W |

Note that the longitudes are found by subtracting the difference in longitude from 151.6° W, the longitude of the first point of tangency.

**4. General Spherical Triangle Solutions.** Of the six parts of the spherical navigational triangle these tables utilize three as entering arguments and tabulate two as respondents. The only remaining part of the triangle is the parallactic (or position) angle, which is the angle between a body’s hour circle and its vertical circle. Values of the parallactic angle, although sometimes used by astronomers, are not essential for navigation, and, in order to keep the tabulations to a minimum, have not been included. Astronomers and other scientists having a requirement for these data have, however, an easy and rapid access to this information through the simple interchange of arguments, thus effecting a complete solution. Application of the following instructions will determine the additional part.

(a) When latitude and declination are of same name, enter the SAME NAME section of the tables with the appropriate local hour angle, with the declination as latitude argument and the latitude as declination argument, and extract the tabular azimuth angle as the parallactic angle.

(b) When latitude and declination are of contrary name, enter the CONTRARY NAME section of the tables with the appropriate local hour angle and with latitude and declination interchanged; the tabular azimuth angle is then the supplement of the parallactic angle (i.e., parallactic angle equals 180° minus the azimuth angle). This method generally requires the availability of all volumes of the series.

An approximate value of the parallactic angle, $X$, accurate enough for most navigational requirements, can be calculated directly from the formula, $\cos X = d/60'$, where d is the difference between successive tabular altitudes for the desired latitude, local hour angle and declination.

The tabular data of these tables include the solution of any spherical triangle, given two sides and the included angle, provided the two sides are regarded as the complements of the latitude and declination, respectively; and the included angle is regarded as the local hour angle; then the complement of the tabular altitude constitutes the third side and the tabular azimuth angle is the angle between the side regarded as the colatitude and the third side; by interchanging the latitude and declination entries, the third angle (position angle) is found as the azimuth angle.

In general if any three parts of a spherical triangle are given, these tables can be used to find the remaining parts; this will sometimes mean searching through the volumes to find, for example, a particular altitude in a particular latitude and a given LHA in order to find the corresponding azimuth angle and declination. The accuracy of such solutions will often be limited to 0.1°, the tabular accuracy of the azimuth angle. Since such solutions are of limited value to the navigator, except for great-circle and composite sailing (see section D.2 and D.3) and require many rules for their use, their complete coverage has been omitted.

## E. BACKGROUND

**1. Accuracy of Tables.** The tabular values as given in these tables have maximum and probable (50%) errors of ±0.05′ and ±0.025′ in altitude and ±0.05° and ±0.025° in azimuth angle.

The maximum error arising from the use of the Interpolation Table for the first-difference correction is ±0.14′, with a probable error of ±0.03′, when used for the interpolation of altitude for declination.

The maximum error arising from the use of the correction for second-differences obtained from the Interpolation Table is ±0.12′ with a probable error of ± 0.03′.

When second differences are completely negligible, the maximum error of an interpolated altitude is ±0.19′ with a probable error of ±0.04′; when the second differences are not negligible and the second- difference correction is included in the interpolation, the maximum error of the calculated altitude will be ±0.31′ with a probable error of ±0.05′.

The largest value of the double-second difference when the value of d is not printed in italics is 3.9′, and if the correction for this value is neglected, an error of up to –0.24′ may be introduced into the computed altitude. But such an error is only possible when the altitude is greater than 60° and when the value of Dec. Inc. is close to 30′. The neglect of the second-difference correction when d is not printed in italics will rarely introduce an error as large as –0.2′.

**2. Production and Printing**. The method used in the production of this publication has bridged the gap between electronic data processing and automatic photocomposing for offset printing.

The calculations were performed on an IBM 1410 electronic computer, using nine significant figures in order to ensure the accuracy of the altitude to a tenth of a minute of arc and the azimuth angle to a tenth of a degree. In addition, all values of the altitude in excess of 88°30′ were recalculated using more appropriate formulas, since determinations of these high altitudes from their sines with only nine figures could introduce errors of the order of 0.0005′, which would sometimes affect the rounding off of the altitude to 0.1′. The recomputation indicated only about three altitudes per volume were thus affected.

The results of the computations for any one volume were produced on magnetic tapes, which were automatically differenced for checking in the direction of increasing declination; manual recalculations also were used in the verification of several quantities on each page.

After the computations were checked, programmed editing instructions were added and the combined data were transcribed onto a fifteen-channel paper tape which was used to operate photocomposing equipment. The data were photoset, character by character, to form a photographic positive of the final pages; photoprints were then examined for completeness and imperfections. All data on each page were punched onto cards, one line to a card, and these cards were compared automatically with the data originally produced by the computer. Before final printing a systematic examination of the proof was made and numerous independent checks were applied.

The Linofilm reproduction procedures and techniques were employed in the reproduction of Volumes 5 and 6; the Linotron method was used for the other volumes of the series.